Development of scheme for solving fluid-structure problem based on loosely coupling method

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Purpose

Development of the scheme for solving a fluid-structure problem on biomechanics

Future

To simulate the interaction between blood wall and blood
Main scheme of fluid-structure coupling problem

- Loosely coupling method
  Fluid dynamics $\rightarrow$ Structure analysis with FDM $\leftarrow$ with dynamic FEM

- Direct coupling method
  CIP method (Cubic-Interpolated Polynomial)
  ALE method (Arbitrary Lagrangian Eulerian)
Basic equations of fluid dynamics

Navier-stokes equations

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} - \mathbf{u}_0) \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u}
\]

Continuity equations

\[
\nabla \mathbf{u} = 0
\]
Discretization with FDM

1. A third-order upwind in convective term

2. A second-order central scheme in other spatial term

3. A first-order Euler explicit scheme for time integrations terms

4. MAC method is used to couple velocity and pressure field.
Basic equations of elastic body (1)

**Kinematic equations**

\[ \sigma_{ij,j} = \rho \ddot{q}_i \]

**Geometric boundary conditions**

\[ q_i = \bar{q}_i \]

**Dynamic boundary conditions**

\[ \sigma_{ij} n_j = \bar{p}_i \]
Basic equations of elastic body(2)

Discretization with dynamic FEM

\[ M\ddot{q} + C\dot{q} + Kq = F \]

Damping matrix

\[ C = \alpha M + \gamma K \]
Direct time integral method

Newmark $\beta$ method

\[ \ddot{q}(t + \Delta t) = \left\{ \frac{\Delta t}{2} \dot{q}(t) + \frac{\Delta t}{2} \ddot{q}(t) \right\} \cdot \left\{ M + \frac{\Delta t}{2} C + \beta \Delta t^2 K \right\}^{-1} \]

- \[ \dot{q}(t + \Delta t) = \dot{q}(t) + \frac{\Delta t}{2} \left[ \ddot{q}(t) + \ddot{q}(t + \Delta t) \right] \]

Velocity

\[ q(t + \Delta t) = q(t) + \frac{\Delta t}{1!} \dot{q}(t) + \frac{\Delta t^2}{2!} \ddot{q}(t) \]

+ $\beta \Delta t^3 \frac{\dddot{q}(t + \Delta t) - \dddot{q}(t)}{\Delta t}$

Displacement

Acceleration
The Velocity of moving grid

\[ u_0 = \frac{x^{n+1} - x^n}{\Delta t} \]
\[ v_0 = \frac{y^{n+1} - y^n}{\Delta t} \]
Flow chart of scheme

- Calculate stiffness matrix
- Calculate mass matrix
- Calculate damping matrix
- Calculate coefficient matrix

- Solve poisson equations
  - FDM
  - Calculate the velocity
  - FEM
  - Calculate the displacement
  - Calculate velocity of moving grid
  - Transfer the grids
Model of simulation

Uniform flow

Elastic body

Rigid body

Point a

90 deg

0.004m

0.038m
## Calculation conditions

<table>
<thead>
<tr>
<th><strong>Fluid analysis</strong></th>
<th><strong>Elastic analysis</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>10,000</td>
<td>0.4</td>
</tr>
<tr>
<td>$\Delta t = 10^{-5}$</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td></td>
<td>11,000 [Pa]</td>
</tr>
<tr>
<td>Newmark method</td>
<td>Damping matrix</td>
</tr>
<tr>
<td>$\beta = 0.25$</td>
<td>coefficient</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 0.02$</td>
</tr>
<tr>
<td></td>
<td>$\gamma = 0.05$</td>
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</tbody>
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**Non dimensional time**

$T = 0.0 \text{ (case 1)}, \ 6.0 \text{ (case 2)}, \ 8.0 \text{ (case 3)}$
Case 1: A pair of vortices

Case 2: Pre Karman vortex row

Case 3: Karman vortex row
Division of domains for fluid and elastic body

The number of grids
182 × 182

Triangular elements
272 elements

One-dimensional interpolation function for displacement
The amplitude at point a in case 1
Generation of traveling wave

Positive pressure

Negative pressure

Elastic body
Vector line of velocity in case 1
Deformation in case 1
The amplitude at point a in case 2
Deformation in case 2
Deformation in case 3

Karman vortex
Conclusion

- We proposed the new scheme based on the loose coupling method for solving a CFS problem.
- The circular cylinder with elastic surface in uniform flow is chosen as the example. We indicate the satisfied results.