The establishment of high-precise one-dimensional numerical simulation model of blood flow for the cardiovascular system.

Tomoki KITAWAKI*,#, Masashi SHIMIZU†, Ryutaro HIMENO#

*Okayama University Medical School,  †Tokyo Institute of Technology,  #Advanced Computing Center, RIKEN

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Outline

To make a computational model of whole body circulatory system

3-D model:
- realistic
- 3D, pulsatile flow fields
- complicated and difficult

1-D model:
- idealized
- wave propagation
- feasible

Modify at junction point

Boundary conditions for 3-D model
Objective

1-D whole body model

Influence of some issues
- Vessel structure (taper, branch, etc.)
- Unsteadiness of blood flow
- Behavior of vessel wall
- Boundary conditions
- Non-newtonian characteristics of blood

Aim:
To establish the high-precise One-Dimensional numerical simulation model

Models:
- Branch angle model
- Unsteady viscous model
- Generalized Viscoelastic Model

Quantitative model

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1: Branch angle
Treatment at branching points

\[
\frac{d Q^1}{d t} = \left\{ \frac{A A^1}{A^1 + A} \left( \frac{(Q^1')^2}{(A^1 + A) / 2} \right) - \frac{(Q^2)^2}{A A^2} \cos \theta_2 - \frac{(Q^3)^2}{A A^3} \cos \theta_3 \right\} + \frac{(P^1 - P) A A^1 / \rho}{\Delta x} \]

cross-sectional area ratio of the tubes: \[
\sum_{A_{\text{output}}/A_{\text{input}}} A
\]
Relationship between the reflected wave and the tube cross-sectional ratio

Fig. 11: Large tube
Discussion and Conclusion

- **1-D computational model of the artery systems**
  - investigation the bifurcation angle dependence
  - a quantitative analysis of the reflected wave

- **The angle effect**
  - the reflected wave at bifurcation point was observed
  - the angle dependence was recognized in large and medium arteries

- **Combination of angle and cross-sectional ratio**
  - peculiar feature of reflected wave
2: Unsteadiness of blood flow
3: Behavior of the vessel wall
One-Dimensional Numerical Model

- **Continuity equation**
  \[
  \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0
  \]

- **Equation of momentum conservation**
  \[
  \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( Q^2 \frac{Q}{A} \right) + \frac{A}{\rho} \frac{\partial p}{\partial x} + f_i = 0
  \]
  - unsteady viscous term

- **Deformation law of the tube**
  \[
  p - p_0 = \frac{1}{C_s} (A - A_0) + f \left( \frac{\partial A}{\partial t} \right)
  \]
  - viscoelasticity of tube wall
  - quasi-steady flow model: \( f_i = 8\pi vV(t) \)
  - approximated unsteady model: \( f_i = c_v 8\pi v \frac{Q(t)}{A(t)} + (c_u - 1) \frac{\partial Q}{\partial t} \)
  - accurate unsteady model: \( f_i = 4\pi v \left\{ 2V(t) + \int_0^t W(t-u) \frac{\partial V(u)}{\partial t} \, du \right\} \)

  \[
  \begin{align*}
  \text{Elastic model} & \quad f = 0 \\
  \text{Voigt model} & \quad f = \frac{hE}{2RA_0} \frac{\partial A(u)}{\partial t} \\
  \text{Generalized Viscoelastic Model} & \quad f = \frac{h}{2RA_0} \int_0^t \sum_{i=1}^n E_i e^{-\frac{(t-u)}{\tau_i}} \frac{\partial A(u)}{\partial t} \, du
  \end{align*}
  \]
Calculation methods of Generalized Viscoelastic Model

- **Unsteady Viscous term**: Kagawa et al. (1983)

\[
f_t = 4\pi v \left\{ 2V(t) + \int_0^t W(t-u) \frac{\partial V(u)}{\partial t} du \right\}
\]

\[
W(t) = \sum_{i=0}^k m_i e^{-n_i (\nu t) / R^2}
\]

\[
y_i = \int_0^t m_i e^{-n_i (\nu R^2 t-u)} \frac{\partial V(u)}{\partial t} du
\]

\[
f_t(t) = 4\pi v \left\{ 2V(t) + \sum_{i=0}^k y_i(t) \right\}
\]

\[
\begin{align*}
y_i(t) &= 0 \\
y_i(t + \Delta t) &= e^{-n_i (\nu \Delta t) / R^2} y_i(t) + m_i e^{-n_i (\nu \Delta t / 2) R^2} \left\{ V(t + \Delta t) - V(t) \right\}
\end{align*}
\]

\[
(t = 0) \quad (t > 0)
\]

- **Viscoelasticity of the tube**

\[
f = \frac{h}{2RA_0} \int_0^t \sum_{i=1}^n E_i e^{-(t-u)/\tau_i} \frac{\partial A(u)}{\partial t} du
\]

\[
W_V(t) = \sum_{i=1}^n E_i e^{-t/\tau_i}
\]

\[
z_i = \int_0^t E_i e^{-(t-u)/\tau_i} \frac{\partial A(u)}{\partial t} du
\]

\[
f = \frac{h}{2RA_0} \sum_{i=1}^n z_i(t)
\]

\[
\begin{align*}
z_i(t) &= 0 \\
z_i(t + \Delta t) &= e^{-\Delta t / \tau_i} z_i(t) + E_i e^{-\Delta t / 2 \tau_i} \left\{ A(t + \Delta t) - A(t) \right\}
\end{align*}
\]

\[
(t = 0) \quad (t > 0)
\]
Experimental apparatus

- 4m silicone tube
- Piston pump
- Flow meter
- Pressure sensors (every 0.7m)
- Water tank
- Valve

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Experimental result
Determination of tube viscoelastic parameter

(a) real part of viscoelastic modulus

(b) loss tangent

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A computational method

- **Computational scheme**
  - **Finite Difference Method**
  - **Space**: 4th order central difference
  - **Time**: Jemson-Baker four stage Runge-Kutta

- **Boundary conditions**
  - **input:**
    - Flow volume
  - **output:**
    - No output flow

- **Initial state**
  - no flow in the tube

### Computational parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional area</td>
<td>0.612 × 10⁻⁴ m² (8.83 mm)</td>
</tr>
<tr>
<td>Input peak pressure</td>
<td>1.5 kPa</td>
</tr>
<tr>
<td>peak flow rate</td>
<td>0.13 m/s</td>
</tr>
<tr>
<td>Max Reynolds number (Re)</td>
<td>1150</td>
</tr>
<tr>
<td>Static Young module (E₀) (wave propagation velocity)</td>
<td>3.05 (MPa) (21 m/s)</td>
</tr>
<tr>
<td>Length of the tube (Δx)</td>
<td>4.0 m</td>
</tr>
<tr>
<td>(Δx)</td>
<td>(0.05 m)</td>
</tr>
<tr>
<td>Total elapsed time (Δt)</td>
<td>4.0 s</td>
</tr>
<tr>
<td>(Δt)</td>
<td>(0.001 s)</td>
</tr>
<tr>
<td>Courant Number (=c Δt/Δx)</td>
<td>0.42</td>
</tr>
</tbody>
</table>

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Comparison between measurement and simulation
Generalized Viscoelastic Model

Experiment:  
Calculation:
Comparison between measurement and simulation
Voigt Model

Experiment:  
Calculation:
Conclusion

- Establishment the treatment of unsteady viscous term and vessel wall viscoelastic term
  - Unsteady viscous model and Generalized Viscoelastic Model can be applied to the deformable tube
  - New calculation method is established.
  - Good agreement with measurement and simulation involving both unsteadiness and visco-elasticity of tube
Future works

- Establishment of the whole body 1-D model
  - Decision of parameters: viscoelasticity of vessel wall
  - Apply to the *in vivo* phenomenon analysis

- Model combination
  - Tree-structured 1-D model and 3-D model

- Verification and validation
  - comparison with 3-D model, experimental results

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