Time-dependent static optimization method to estimate muscle force from the motion

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Abstract

Static optimization method that is based on inverse dynamics is the most popular and easiest way to estimate muscle force from the motion data, and therefore it has been used to analyze various human motion data in biomechanics. These methods are all based on time-dependent optimization methods that do not take into account the whole motion. In this paper we propose a time-dependent, global optimization method for predicting the muscle force.

The idea is to optimize the objective function based on the history of muscle activation through the motion while using the muscle force - joint torque relationship as linear equality constraints, and muscle activation dynamics that limit sudden increase or decrease of the muscle force as inequality constraints. Although the overhead of our method is heavier comparing to previous time-independent methods, the solution can be obtained using quadratic programming in real time.

Since our method is a time-dependent method, it can be applied to motions such as jumping or running, that could be handled only by dynamic optimization methods.

1 Introduction

The problem to estimate the muscle force through the motion has been a major topic in biomechanics for a long time. One of the oldest and most popular method used today is based on inverse dynamics: that is to first calculate the joint torques from the motion data, and then estimate the muscle force from the torque. Since the number of muscles is greater than the degrees of freedom of the body, optimization methods are mainly used to estimate the muscle force from the joint torques. In the old days, some researchers proposed to use linear programming for such purpose [3, 9]. Linear programming methods did not work well because in many cases the muscles exerted either the maximum or minimum force exertable. Later researchers set nonlinear objective functions as the criteria, and solved the optimization problem using various techniques such as composite gradient method [10], pseudo-inverse method [5], and sequential quadratic programming method [2]. Some researchers took into account muscle dynamics in order to limit the muscle force.

These researches are classified as time independent static optimization methods today. This is because the muscle forces at each moment are calculated only using the joint torques at that moment. The problem with static optimization techniques is that it can be applied only to motion such as bipedal gait with which activaton dynamics of the muscle do not crucially affect the motion [2].

Today, dynamic optimization methods are considered as superior methods to estimate the muscle force. Dynamic optimization methods are based on activation dynamics of the muscles and simulation using forward dynamics. The muscles are activated by twitch signals, and the body model is controlled by the muscles. The final muscle-activation history is obtained by minimizing an objective function that takes into account the whole motion. It has been applied to motion such as maximal vertical jumping [7, 8], bipedal gait [6, 1], and etc.

The problems with dynamic optimization methods include (1) the difficulty to obtain the initial motion to start the optimization and (2) the heavy computational cost. The first problem makes it difficult to apply dynamic optimization techniques to arbitrary motions. The researcher must somehow first guess the initial muscle twitch signals that realize the inital motion to start the optimization.

And mentioning about the second problem, it is reported that dynamic optimization techniques costs thousands of times that by static optimization methods. In addition to that, it is also reported that static and dynamic optimizations are almost equivalent for motion such as bipedal gait [2]. However, it is known that they are not equivalent for motion such as maximal vertical jumping or sprinting with which the activation dynamics of the muscles are playing a crucial role. Therefore, for analysis of such motion, dynamic optimization is considered the only precise method today.

In this paper we propose a time-dependent, global optimization method for predicting the muscle force that is based on inverse dynamics. The method is similar but different from previous static optimization techniques.

The idea is to optimize the objective function based on the history of muscle activation through the motion while using the linear relationship between the muscle force and joint torques as linear equality constraints, and muscle activation dynamics that limit sudden increase or decrease of the muscle force as inequality constraints. Although the overhead of our method is heavier than previous time-independent methods, as all the equality and inequality constraints are set linear, the solution can be obtained using quadratic programming in real time. Since our method is a time-dependent method, it can be applied for motions such as jumping or running, that could be handled only by dynamic optimization methods.

2 Methods

We have used the musculoskeletal model by Delp *et al* [4] for the motion analysis. Each musculotendon is based on Hill's muscle model. The muscle model and their parameters are also from Delp *et al* [4].

The model is composed of three elements: the contractile element (CE, muscle fibers), the parallel elastic element (PEE, connective tissue around the fibers and fiber bundles), and the series elastic element (SEE, muscle tendon).

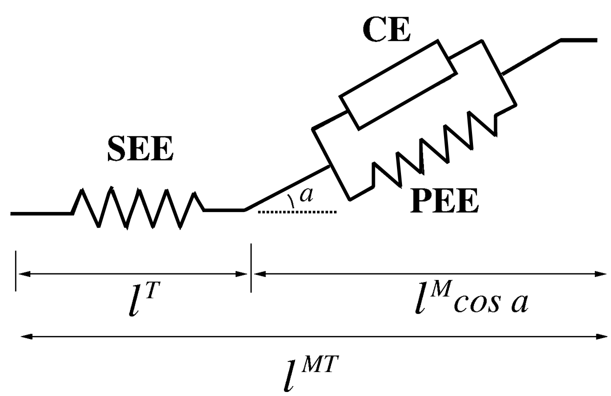


Figure 1: The muscle model by Hill used in this study

At each moment of the motion, as the musculotendon length is determined by the posture, the minimum force that must be exerted by the muscle can be calculated:

$$F_m^{min}(t) \leq F_m(t) \tag{1}$$

where $F^m(t)$ is the musculotendon force of the *m*th muscle, and $F_m^{min}(t)$ is the minimum force by this muscle when its activation level is 0.

After the trajectory of the body has been determined, the torque exerted at each joint is calculated using inverse dynamics. The torque $\tau_{\mathbf{j}}(t)$ exerted at joint j, is generated by the muscles crossing the joint:

$$\tau_{\mathbf{j}}(t) = \sum_{m} F_m(t) r_{m,j}, j = 0, ..., n_{dof} \tag{2}$$

where $r_{m,j}$ is the moment arm of muscle m abouth the jth joint axis, and n_{dof} is the total degrees of freedom whose torque is generated by the muscles.

Previous time-independent methods estimated the muscle force at each moment by optimizing functions such as

$$J = \sum_{m}^{n_m} a_m(t_i)^2 \tag{3}$$

where $a_m(t_i)$ is the activation level of muscle m, and n_m is the total number of muscles.

Instead of solving such static optimization problem at each time step, we propose a new method to solve a single optimization problem through the motion:

$$J = \sum_{i}^{n_s} \sum_{m}^{n_m} a_m(t_i)^2 \tag{4}$$

where n_s is the number of time steps. The muscle force - joint torque equations (2) are used as equality constraints. The amount of each muscle force is limited by equation (1).

The activation dynamics of the muscle used in this paper is not fully physiological. This is because most of the equations of muscle dynamics are nonlinear: such as the twitching of muscle activations, force - length relationships of the muscles, and etc. As a result, without linearlizing these relationships, solving for the muscle forces become a nonlinear problem, that is hard to be solved.

First, the muscle activation level is calculated from the muscle force using the following equation:

$$a_m = \frac{F_m - F_m^{min}}{F_m^{max} - F_m^{min}} \tag{5}$$

where F_m^{max} is the static maximum force exerted by the muscle when its activation level is 1, and when the contraction velocity v_m is 0.

Next, the muscle activation dynamics, that limit the change of activation level of the muscles between the time steps, is defined by the following equation:

$$\Delta a_{min} \leq a_m(t_{i+1}) - a_m(t_i) \leq \Delta a_{max}, i = 0, 1, .., n-1 \tag{6}$$

where Δa_{max} and Δa_{max} are constant values that limit the change of the muscle activation.

In summary, the problem to be solved here can be written by the following form:

$$\min_{F^m} \sum_{i}^{n_s} \sum_{m}^{n_m} (\frac{F_m(t_i) - F_m^{min}(t_i)}{F_m^{max}(t_i) - F_m^{min}(t_i)})^2 \tag{7}$$

where

$$\tau_j(t_i) = \sum_m F_m(t_i) r_{m,j}(t_i) \tag{8}$$

$$F_m^{min}(t_i) \leq F_m(t_i) \tag{9}$$

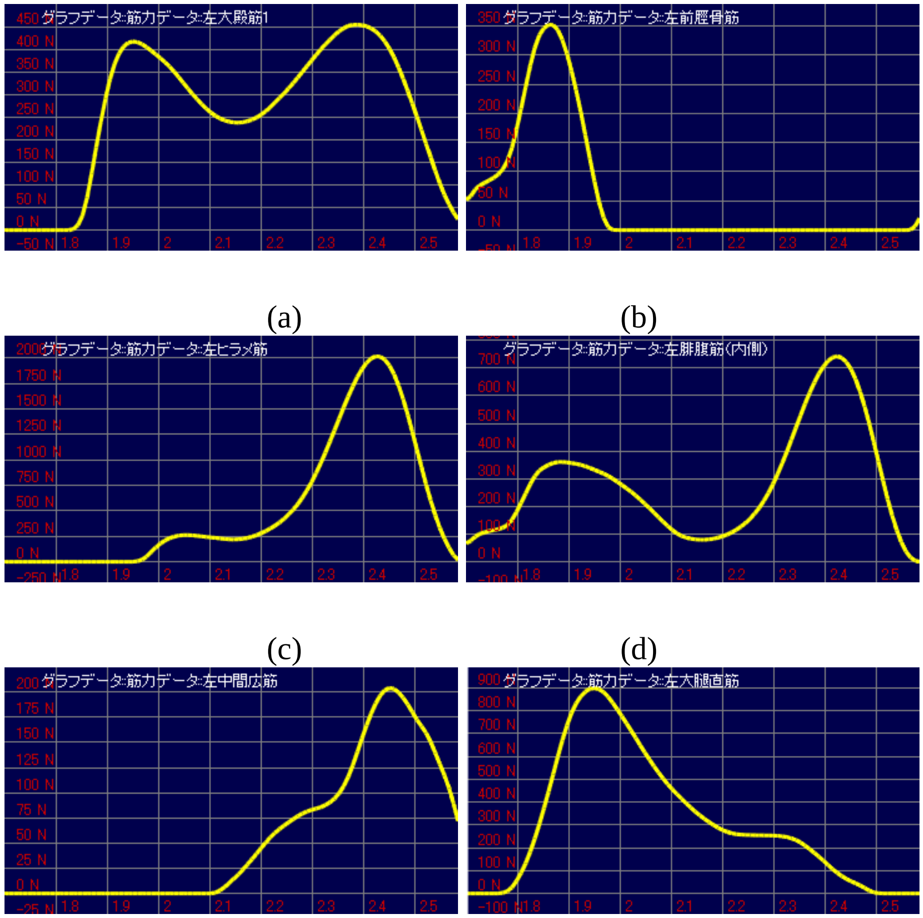
$$\Delta a_{min} \leq \frac{F_m(t_{i+1}) - F_m^{min}(t_{i+1})}{F_m^{max}(t_{i+1}) - F_m^{min}(t_{i+1})} - \frac{F_m(t_i) - F_m^{min}(t_i)}{F_m^{max}(t_i) - F_m^{min}(t_i)} \leq \Delta a_{max}. \tag{10}$$

This problem is composed of $n_s \times n_m$ variables, $n_{dof} \times n_m$ equality constraints and $n_s \times n_m + 2 \times n_s \times n_m$ inequality constraints. Although the number of variables is large, since the matrix of the objective function and constraints are both sparse, this problem can be a solved using quadratic programming in real time.

3 Results

To adapt the musculoskeletal model to the subject to whom we apply our inverse dynamics method, the size of each body segment was linearly scaled. Muscle parameters such as the origin and insertion of the muscles, fiber and tendon length were scaled as well. The new inertia and mass of the body segments were calculated using the scaling factors.

The optimization method proposed in this research was applied to a gait motion. The activation history of the major muscles used for a gait motion during the support phase are shown in figure 3. We have compared our results with the EMG data, and found out the pattern matched very well



(e) (f)

Figure 2: The muscle activation history by the (a) gluteus maximus (b) tibialis anterior (c) soleus (d) gastrocnemius (e) vastus medialis, and (f) rectus femoris.

with the muscle activation calculated using our method. For example, using our method, we could observe the activation of the tibialis anterior muscle before its landing to the ground. Such kind of activation could not be calculated using previous static optimization methods.

4 Summary

In this paper we have proposed a time-dependent, global optimization method for predicting the muscle force.

The idea was to optimize the objective function based on the history of muscle activation through the motion while using the muscle force - joint torque relationship as linear equality constraints,

We have applied our method to a gait motion and obtained a result that match well with EMG data.

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