A two-dimensional thermal model for determining cold-stressed effects on a human finger

Ying HE[1]

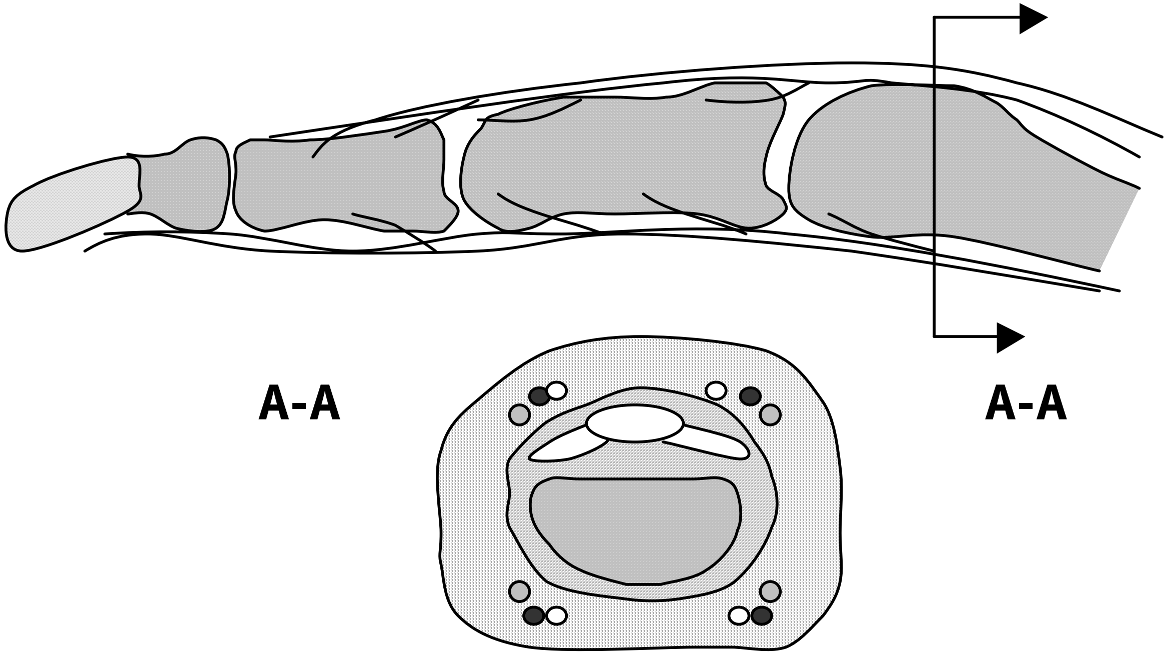
Computer and Information Division, RIKEN

Tetuya KAWAMURA[2]

Graduate School of Humanities and Sciences, Ochanomizu University

Ryutaro HIMENO[3]

Computer and Information Division, RIKEN



Arteries and the cross section structure of a finger
(cited from Tachenatlas Der Anatomie)

Research Background

• Investigate the relationship between finger skin temperature and digital blood flow

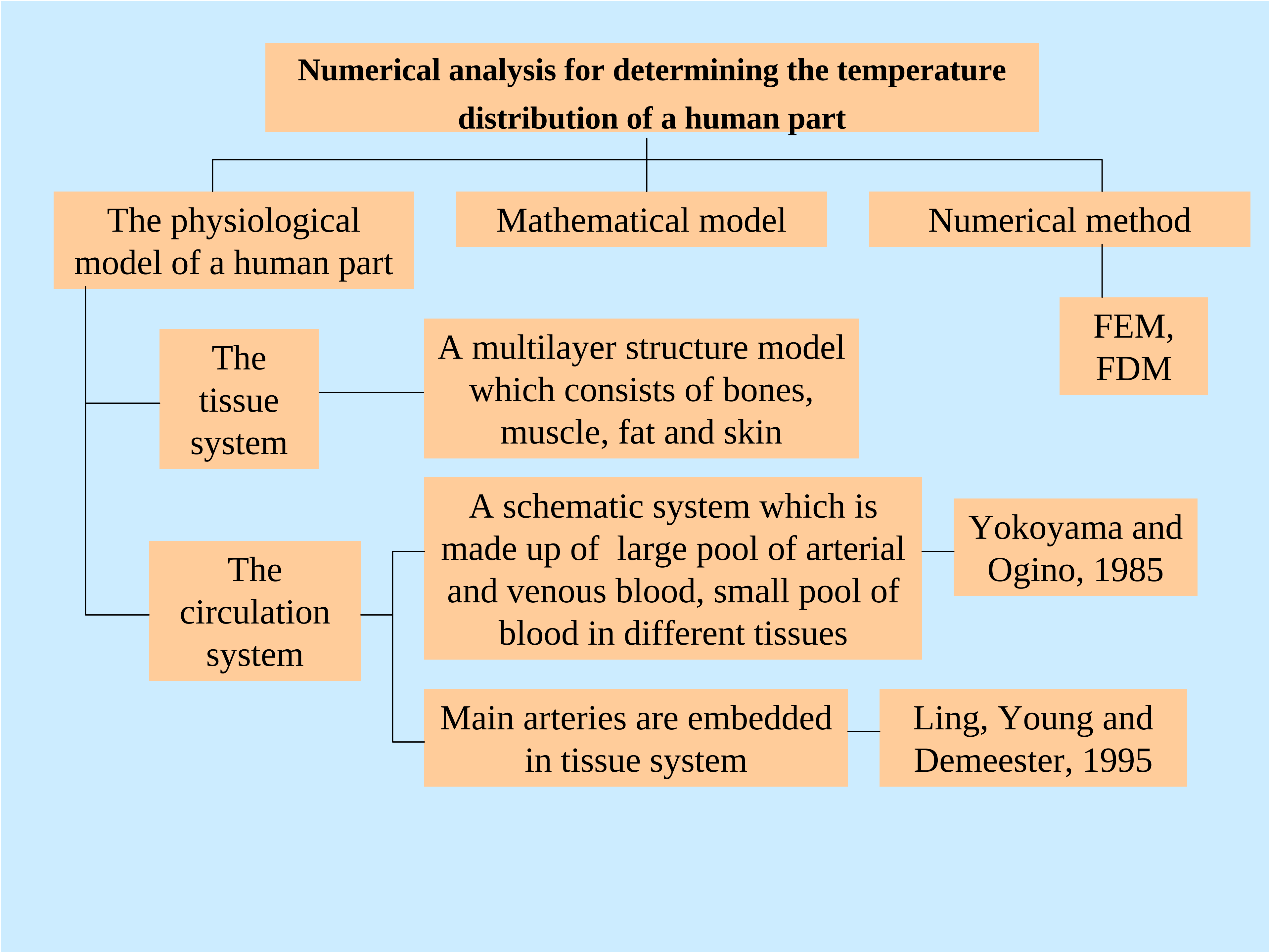
a. In diagnostic application:

Finger skin temperature is widely used as a parameter for the disappearance of cold-induced vasoconstriction (e.g. the study of Raynaud’s syndrome)

b. In industrial application:

How to maintain hand dexterity and comfort , how to avoid cold injury during cold exposure when hand work is involved

• The first step to build 3D thermal-fluid model of hand


Numerical analysis for determining the temperature distribution of a human part
The physiological model of a human part
Mathematical model
Numerical method
FEM, FDM
The tissue system
A multilayer structure model which consists of bones, muscle, fat and skin
The circulation system
A schematic system which is made up of large pool of arterial and venous blood, small pool of blood in different tissues
Yokoyama and Ogino, 1985
Main arteries are embedded in tissue system
Ling, Young and Demeester, 1995

Mathematical model

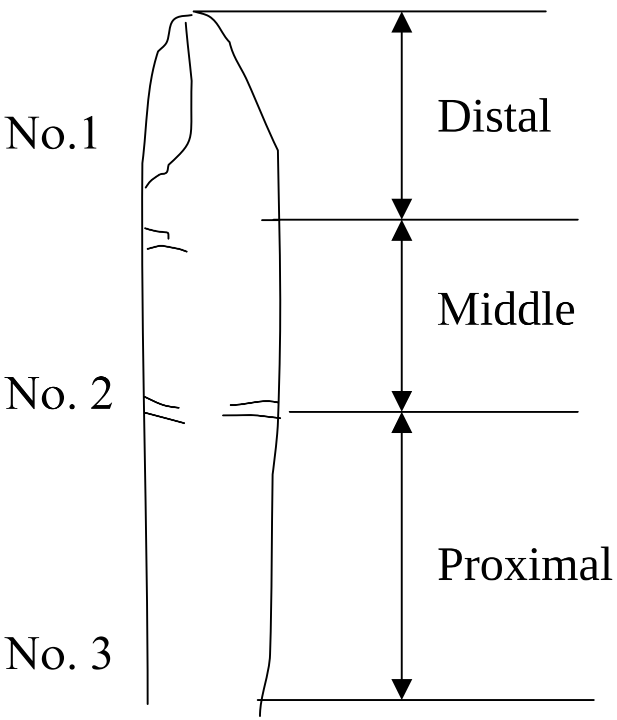
Paired-vessel model

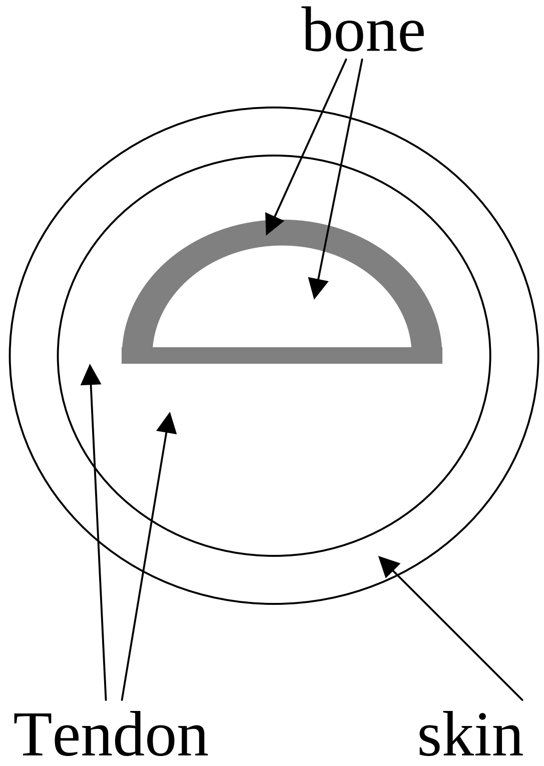
- By Keller and Seiler (1971)
- The heat transport process is described by the three equations (heat balance equations about tissue, artery and vein)
- In Yokoyama and Ogino's analysis (1985)

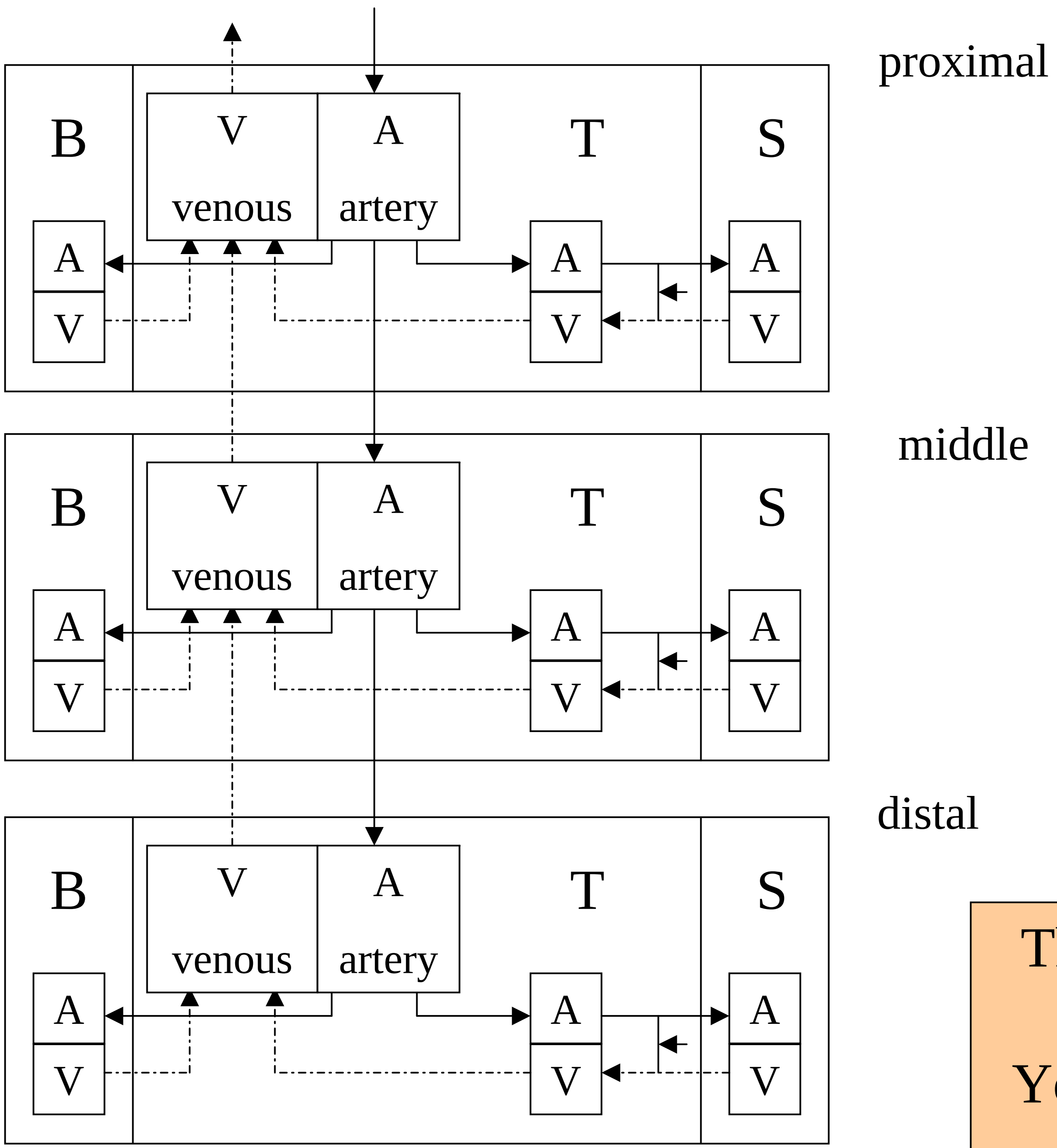
Pennes bioheat equation

- Adding the warming effect of blood flow into the heat conduction equation. The blood acts as a heat source.
- In Ling, Young and Demeester's analysis (1995)

Modeling on the shape and internal structure of fingers (Yokoyama and Ogino, 1985)







The vascular system of middle finger in Yokoyama and Ogino's model

Paired-vessel model

Tissue:

$$\rho_t c_t \frac{\partial T_t}{\partial t} = \nabla(k_t \nabla T_t) + \frac{fc_b}{V}(T_a - T_t) + H_a(T_a - T_t) + H_v(T_v - T_t) + q_{met}$$

Artery

$$\rho_b c_b V_a \frac{\partial T_a}{\partial t} = f_a c_b (T_{am} - T_a) + \int_v H_a (T_t - T_a) dV + H_{av}(T_v - T_a)$$

Vein

$$\rho_b c_b V_v \frac{\partial T_v}{\partial t} = f_v c_b (T_{vn} - T_v) + \int_v (\frac{fc_b}{V} + H_v)(T_t - T_a) dV + H_{av}(T_a - T_v)$$

f : blood flow rate; *V*:volume; *H*: heat transfer rate; *c*:specific heat; ρ: density;

b: blood; *t*: tissue; *a*: artery; *v*: vein; am: adjacent artery system; *vn*: adjacent venous system; *av*: exchange between arterial and venous blood pool

The outline of the present research

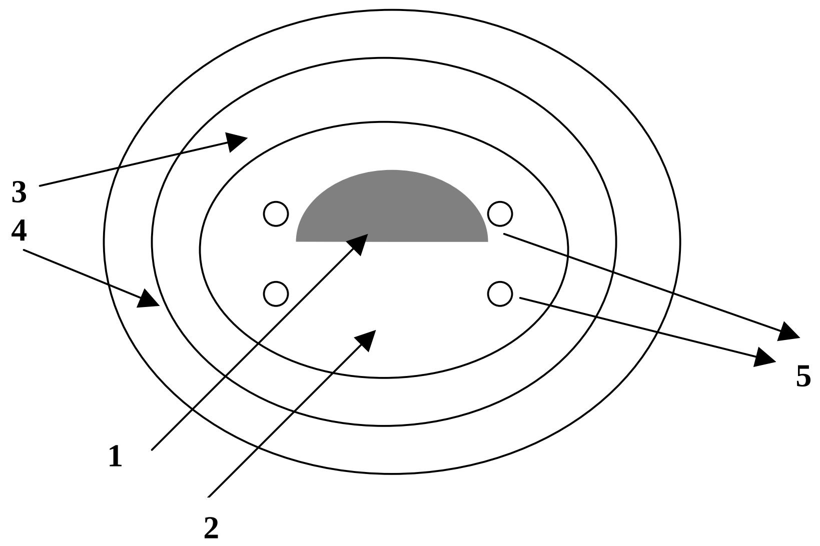
- **Pennes model (1948)**
- **The physiological model is developed from the model of Yokoyama and Ogino (1985) and the model by Ling et al. (1995)**
- **Simulated conditions:**

(1) finger in the air

(2) ice water immersion of the finger with normal blood flow

- **Numerical Method:**

FDM (finite difference method)



1. Bone 2. Tendon 3. Dermis 4. Epidermis 5. Artery

The geometrical model of the middle finger

Pennes bioheat equation (1948)

$$\rho_t c_t \frac{\partial T_t}{\partial t} = \nabla(k_t \nabla T_t) + q_{met} + \omega \rho_b c_b (T_a - T_v)$$

$\rho_t c_t$ volumetric specific heat of tissue

q_{met} metabolic heat generation

$\rho_b c_b$ volumetric specific heat of blood

k_t thermal conductivity of tissue

ω blood perfusion rate

T_a temperature of artery

T_v temperature of vein, assumed to be equal to the local tissue temperature

Boundary conditions

1. Tissue temperature in the deep

$$r = 0 \qquad \frac{\partial T_t}{\partial r} = 0$$

2. Surface temperature of the finger

a. in cold water:

$$r = R, \qquad \lambda_r \frac{\partial T_t}{\partial r} = h_{cw}(T_{sur} - T_w)$$

h_{cw}: convective heat coefficient between skin and water

b. in air:

$$r = R, \qquad \lambda_r \frac{\partial T_t}{\partial r} = h_{ca}(T_{sur} - T_a) + h_{ra}(T_{sur} - T_r) + E_{diff}$$

E_{diff}: Evaporative heat loss from the skin

h_{ca}: convective heat coefficient between skin and air

h_{ra}: radiant heat coefficient between skin and air

3. The heat transfer between main artery and tissues, The artery temperature is assumed to be 37^{o}C . Fully developed laminar flow inside tube

$$r = r_{artery\,\min} \ or \quad r = r_{artery\,\max} \quad \lambda_r \frac{\partial T_t}{\partial r} = h_{at}(T_t - T_{artery})$$

$$\theta = \theta_{artery\,\min} \ or \quad \theta = \theta_{artery\,\max} \quad \lambda_\theta \frac{\partial T_t}{r\partial \theta} = h_{at}(T_t - T_{artery})$$

4. Initial temperature :

- Assumed to be 32 ~ 34 ^{o}C in every place of the finger

Closure Relationship

1. Evaporative heat loss

$$E_{sk} = h_e(P_s - P_a)$$

P_a water vapor pressure in ambient air, kPa

$P_{sk,s}$ water vapor pressure at skin, normally assumed to be that of saturated water vapor at t_{sk}, kPa

h_e evaporative heat transfer coefficient (analogous to h_c), W/m^2 •kPa

$$h_e = 124\sqrt{V} \qquad W/m^2\,kPa$$

2. Convective heat transfer coefficient

$$h_{ca} = 4.0 \qquad W/m^2K \qquad 0 < V < 0.15 \qquad m/s$$

3. Radiative heat transfer coefficient

for typical indoor temperature, h_r is nearly constant

$$h_r = 4.7 \qquad W/m^2K$$

4. Metabolic heat production

the typical metabolic rate for an average adult

5. Heat transfer coefficient between water and tissue

free convection around horizontal cylinder

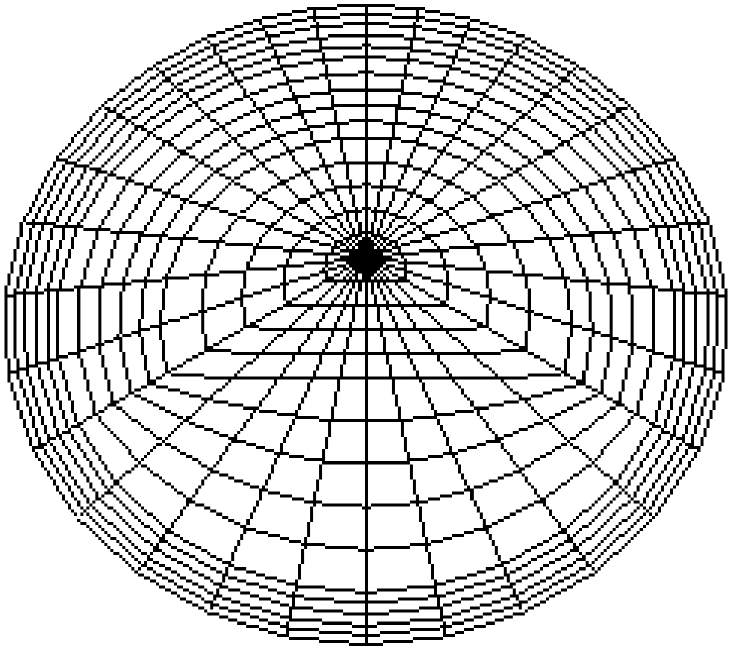
6. Fully developed laminar flow inside the main artery.

Nusselt number of blood

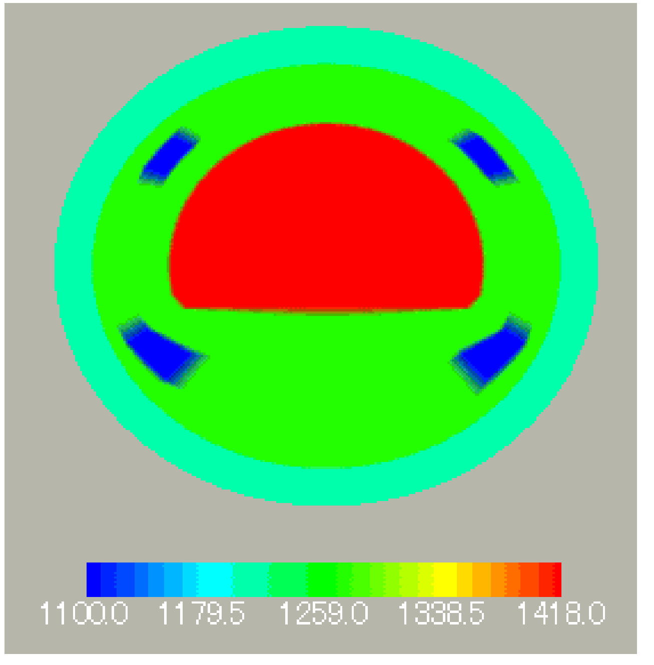
$$Nu_D = 3.66$$

Thermal properties of tissues

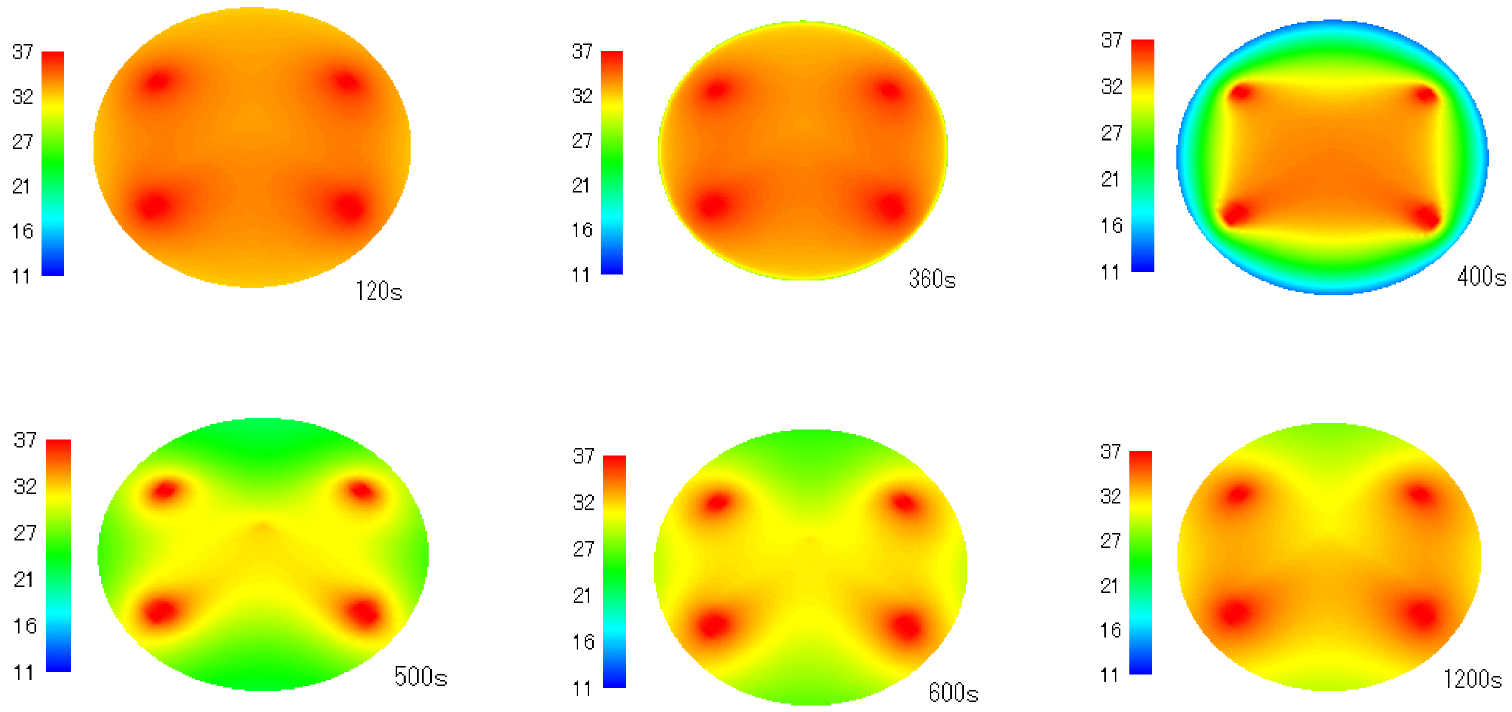
	bone	tendon	dermis	epidermis	blood
$\rho(kg/m^3)$	1418	1270	1200	1200	1100
$c(J/kgK)$	2094	3768	3391	3391	3300
$\lambda(W/mK)$	2.21	0.35	0.53	0.21	0.5
$\omega(ml/ml/min)$	2.0/100	3.43/100	24/100	0	



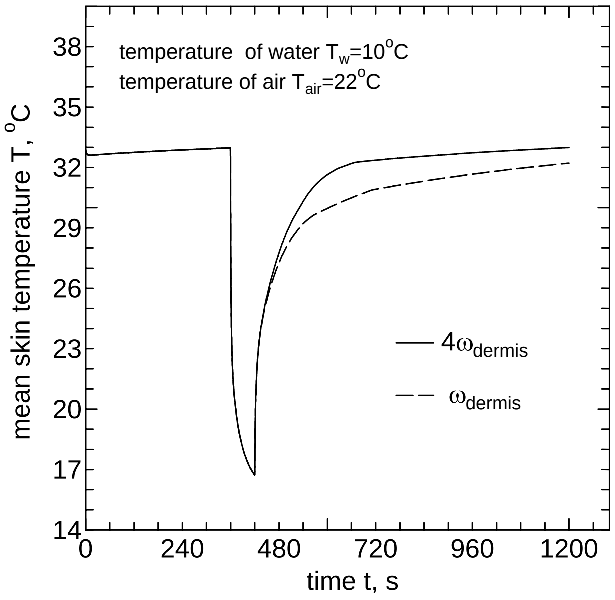
Grid in the cross section of finger

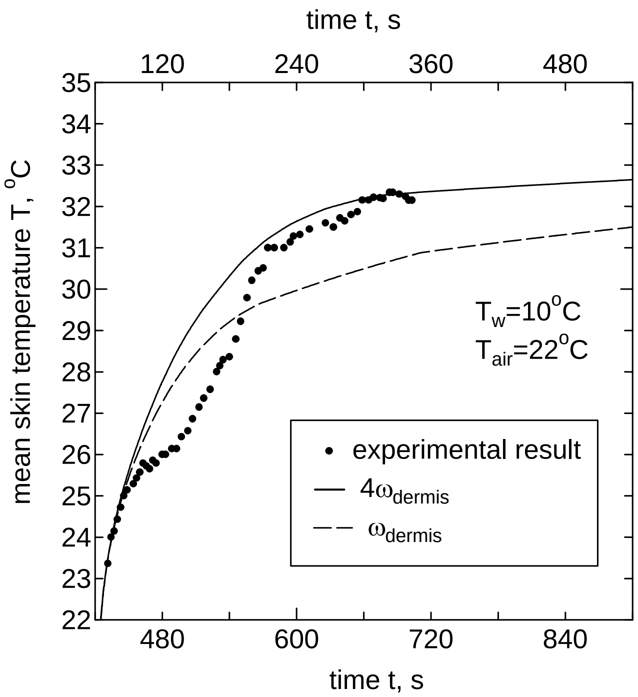


Density of different tissues in the middle finger



Temperature Distribution of the Middle Finger in Air and Cool Water





The variation of mean skin temperature

(a. simulated results b. the comparison between simulated results and the experimental result)

Summary

- 2D temperature distribution was obtained based on Pennes bioheat equation
- The blood flow rate in dermis become larger after cold stimulus
- The re-warm speeds are different around the finger. The side part re-warms faster.

Present work

- 2D numerical analysis to investigate the effect of blood flow on the fingers (with M. Shirazaki)

a FEM coupling model of blood flow in vessels and heat transfer in tissues

- The experiment to measure finger-tip temperatures and blood flow rate

a. Using ultrasound doppler flow meter to measure velocity of finger blood flow

b. Using thermocouples and thermograph to measure finger skin temperature in different conditions