NUMERICAL STUDY ON THE RELATIONSHIP BETWEEN THE FLOW RATE AND TEMPERATURE IN A PERIPHERAL ARTERY SIMULATED BY A ONE-DIMENSIONAL MODEL OF AN ELASTIC TUBE

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ABSTRACT

Blood flow plays an important role of heat transfer in living tissue; in particular, the flow rates in peripheral arteries of the hand are closely related to the fingertip temperature. A study of the haemodynamic effects on temperature may enhance our understanding of the mechanism for thermoregulation and of such circulation diseases as Raynaud's syndrome. We have developed a one-dimensional elastic tube model for simulating the blood flow and temperature in a peripheral artery. This model is based on the one-dimensional flow equations of continuity, momentum and state, and the energy equation. The energy equation for the elastic tube is based on the Keller and Seiler model and is different from that for a rigid tube in that the cross-sectional area of the blood vessel changes with time and space.

The results show that the pulsating axial flow rate produces a pulsating temperature. Moreover, the arterial temperature response follows closely the change in flow rate. This implies that the temperature of the peripheral vessel may significantly depend on the variation in flow rate.

Although further research is necessary in modeling the human systemic circulation (arteries, capillaries, and veins), it is hoped that this model may be applicable to medical diagnosis. The combination of this model with the thermal model for solid tissue is expected.

Nomenclature

A	cross-sectional area of artery, m^2
c	specific heat, J/kgK
E	Young's modulus, kg/s^2m
h	wall thickness of blood vessel, m
L	length of blood vessel, m
p	pressure, Pa
q	flow rate, m^3/s
R_T	total resistance of terminal beds, Nsm^{-5}
r	radius, m
t	time, s

u	velocity in x direction, m/s
x	axial length coordinate, m
δ	thickness of the boundary layer, m
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3
ω	blood perfusion rate, m^3 of blood/s/m^3 of tissue
Δt	time increment, s
Δx	spatial increment, m

Subscripts

0	initial state
b	blood
bot	bottom
t	top
max	maximum value
in	inlet parameters

Introduction

The body temperature depends on the circulation system and the various aspects of its control mechanism. On the other hand, the environmental temperature also affects the action of the heart and blood vessels. Thus, a study on the relationship between the blood flow rate and temperature is important. A number of experimental studies have reported that the flow rates in peripheral vessels of the hand were closely related to the fingertip temperature; for example, the haemodynamic changes while smoking and from mental stress affect the fingertip temperature (8,9). The thermal regulation ability is different between men and women due to the different controlability of blood flow (10).

In the authors' previous study (1), a two-dimensional finite element thermal-fluid model was built up to investigate the effect of blood flow on the temperature distribution of a finger. The finger consists of countercurrent major arterial and venous blood vessels, bone, tendon and skin. We assumed that the blood vessels were rigid, and the basic Navier-Stokes equations and energy equation were employed to describe the behavior of the blood flow and the solid tissues. The computed results show that the skin temperature decreased with decreasing blood flow velocity. However, the degree of temperature variation with differing blood flow velocity was quite small, implying that the effect of the cross-sectional area of a blood vessel should be used in investigating the relationship between the blood flow rate and temperature in the peripheral circulation system. The development of a one-dimensional thermal-fluid model of a blood vessel that incorporates the effects of blood flow, transmural pressure, cross-sectional area, and elasticity of the blood vessel would therefore be valuable.

Numerous models have been presented to analyze the pressure and flow waveforms for the whole human body. Among them, the structured-tree model of systemic arteries by Olufsen *et al*. (2) may be the latest one; they built up a systemic artery tree based on magnetic resonance measurements and statistical relationships. The blood flow in the larger system arteries was modeled by using one-dimensional equations derived from the axisymmetric Navier-Stokes equations for flow in an elastic tube. The small arteries and arterioles were modeled by using derived linearized governing equations that are applicable to calculating the root impedance of the structured tree. The blood flow and pressure were computed as functions of time and axial distance within each of the arteries. The computed blood flow and pressure in every artery showed favorable agreement with the magnetic resonance measurements.

In respect of the mathematical model for bioheat transfer, Keller and Seiler (3) have presented a model that considered the heat transfer between separate tissue, artery, and vein compartments. They derived one-dimensional steady-state energy equations for the arteries, veins, and tissues. The cross-sectional area of each blood vessel is considered to be constant in their model.

This present paper presents the results of an initial study on the effect of blood flow in a single compliant vessel on the resulting blood temperature distribution. The one-dimensional energy equation in a compliant vessel is based on the Keller and Seiler analysis method. The blood flow rates and cross-sectional areas that are necessary in the energy equation of a compliant vessel are computed from the model of Olufsen *et al.* (2). The two-step Lax-Wendroff method is employed in computing the flow rate and cross-sectional area, and the upwind method is used to transform the energy equation into and algebraic form. The blood temperature at different inflow conditions is then obtained. It is believed that the results of this study will be helpful in understanding the mechanism for thermal regulation in the peripheral circulation system.

Model

Predicting temperature of blood flowing in a compliant vessel requires four equations: two equations ensure the conservation of mass and the conservation of momentum, one ensures the conservation of elasticity, and the forth equation represents the conservation of energy. The continuity equation and momentum equation are as follows:

$$\frac{\partial A}{\partial t}+\frac{\partial q}{\partial x}=0 \tag{1}$$

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{A}\right)+\frac{A}{\rho}\frac{\partial P}{\partial x}=-\frac{2\pi\nu R}{\delta}\frac{q}{A} \tag{2}$$

The state equation which represents the conservation of elasticity is written as

$$P(x,t)-P_0=\frac{4}{3}\frac{Eh}{r_0}\left(1-\sqrt{\frac{A_0}{A}}\right) \tag{3}$$

The relationship between Young's modulus, the vessel radius, and the wall thickness is expressed empirically by

$$\frac{Eh}{r_0}=k_1\exp(k_2 r_0)+k_3 \tag{4}$$

where k_1=2.00×10^{7}g/(s^{2}cm), k_2=-22.53cm^{-1}, and k_3=8.65×10^{5}g/(s^{2}cm).

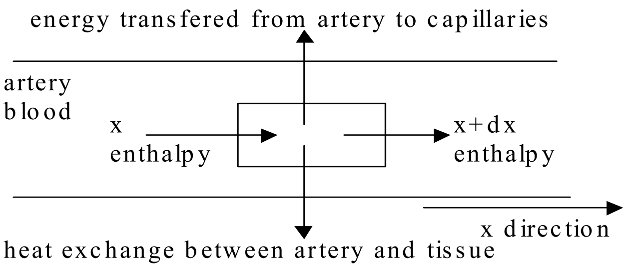

Figure 1. Schematic view of heat exchange between an artery and the surroundings

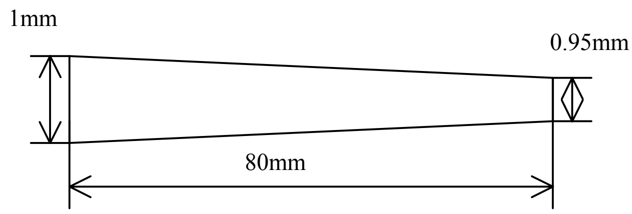

Figure 2. A simulated artery vessel

The momentum equation can be expressed as a function of q and A by substituting state equation 3 into equation 2, such that

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{A}+B\right)=-\frac{2\pi\nu R}{\delta}\frac{q}{A}+C \tag{5}$$

where

$$B=\sqrt{\pi A}\frac{1}{\rho}\frac{4}{3}Eh$$

$$C=\sqrt{\pi A}\frac{8}{3}\frac{1}{\rho}\frac{\partial}{\partial x}(Eh)-\frac{4}{3}\frac{A}{\rho}\frac{\partial}{\partial x}\left(\frac{Eh}{r_0}\right) \tag{6}$$

The energy equation for the elastic vessel is based on the Keller and Seiler model (3). As shown in figure 1, the energy balance equation for the arterial element can be written as

$$\begin{aligned}\frac{\partial(\rho_b A c_b T_b)}{\partial t}&=-\frac{\partial(\rho_b A u c_b T_b)}{\partial x}\\&\quad-\omega\rho_b c_b A T_b-h_{art}A_s(T_b-T_t)\end{aligned} \tag{7}$$

Since the density and specific heat are assumed to be constant, and the blood flow rate can be expressed as $q=Au$, equation 7 can thus be rewritten as

$$\begin{aligned}\rho_b c_b\frac{\partial(AT_b)}{\partial t}+\rho_b c_b\frac{\partial(qT_b)}{\partial x}\\=-\omega\rho_b c_b A T_b-h_{art}A_s(T_b-T_t)\end{aligned} \tag{8}$$

Different from the Keller and Seiler model, the cross-sectional area varies with time and space. Equation 8 reveals that the energy change in a unit time and distance equals the energy transferred from the artery to the capillaries and tissues.

The numerical model is first applied to a single vessel. An artery running inside the finger is selected as illustrated in figure 2. The vessel is assumed to taper exponentially as

$$r_0(x)=r_t\exp\left(\log\left(\frac{r_b}{r_t}\right)\frac{x}{L}\right) \tag{9}$$

At the inlet to the artery, the flow rate is specified in the form of the physiological volumetric flow rate which is expressed as

$$\begin{aligned}q_{in}&=q_{\max}(0.251+0.290(\cos\Phi+0.97\cos 2\Phi\\&\qquad+0.47\cos 3\Phi+0.14\cos 4\Phi)\\\Phi&=2\pi t-1.4142\end{aligned} \tag{10}$$

Maximum flow rate q_{max} was obtained from the measured value by an ultrasound Doppler probe. The outflow boundary condition is determined from a pure resistive model, the relationship between the pressure and flow being simply written as

$$p(x,t) - p_0 = R_T q(x,t) \tag{11}$$

The inflow blood temperature is assumed to be 37°C, despite the temperature difference between the aorta and smaller arteries.

Equations 1 and 5 are transformed into an algebraic form by the two-step Lax-Wendroff method. The difference equations are second-order-accurate in space and time. The blood temperature can be obtained from the discretized form of equation 8, in which the upwind scheme was used for the convective term.

The stability criterion for the linearized equations is

$$\Delta t \le \frac{\Delta x}{c}, \quad c = \left| \frac{q}{A} \pm \sqrt{\frac{A}{\rho}\frac{\partial p}{\partial A}} \right| \tag{12}$$

where c is the wave propagation velocity. Hence, Δt in this model is taken as less than 1×10^{-5}s.

Results and discussion

The parameters used in the numerical analysis are listed in Table 1.The predicted blood temperatures for different input flow rates are shown in Figure 3(a). It can be seen that, with the flow rate decreasing, the blood temperature accordingly decreases. If the input flow rate is quite small, the blood temperature can considerably decrease. Moreover, the temperature varies periodically due to the pulsating blood flow rate and cross-sectional area. For comparison, the corresponding pressure variations were also plotted as shown in Figure 3(b). The pressure also decreases with the input flow rate.

Figure 4 gives the spatial and temporal variations in the pressure and temperature during one period. It can be seen that the blood temperature decreases along the axial direction.

The temperature variation with the input flow rate in the transient state was also investigated. The inflow rate was initially at the normal level, before decreasing to a lower value after 5 minutes. The duration of this inflow rate was 5 minutes. Finally, the inflow rate recovered to the initial level. The inflow rate cycle is shown in Figure 5(a), and the corresponding temperature variation is plotted in Figure 5(b). As can be seen from this figure, as the inflow rate decreases, the temperature correspondingly decreases. The difference in the plots at the different axial positions is due to the assumption that the temperature at the inlet is constant.

Table 1. Parameters used in the numerical analysis

R_T (Nsm^{-2})	0.5280×10^{10}
ν (m^2/s)	4.6×10^{-6}
ρ_b (kg/m^3)	993
c_b (Jkg/K)	3300
h_{art} (W/m^2K)	1800
ω (m^3 of blood/s/m^3 of tissue)	0.0005-0.0008

Concluding remarks

The numerical analysis by a one-dimensional model was conducted to investigate the effect of blood flow on the blood temperature. The waveform flow rate, transmural pressure, and elasticity of the vessel are each considered in this model. It was found that the arterial temperature response closely followed the change in blood inflow flow rate which was not obvious from the results of our previous study (1). This improvement resulted from the effects of both the blood velocity and of

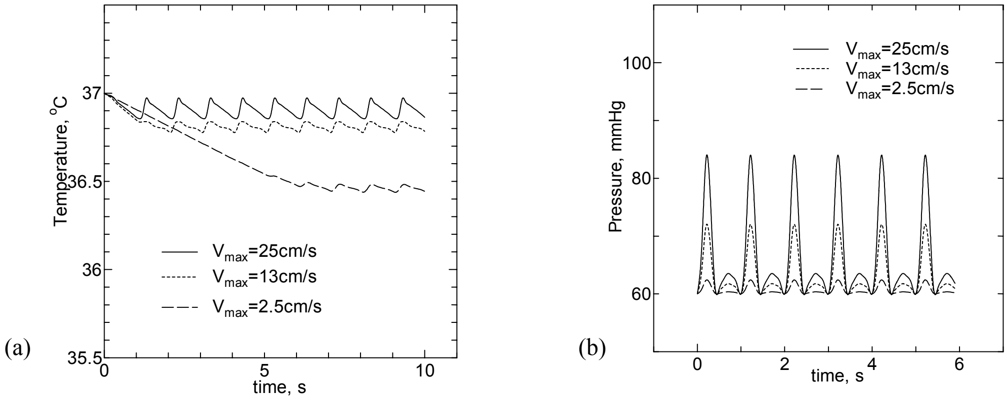

Figure. 3 The temporal variations of artery temperature
(a) and pressure (b) at different inflow conditions

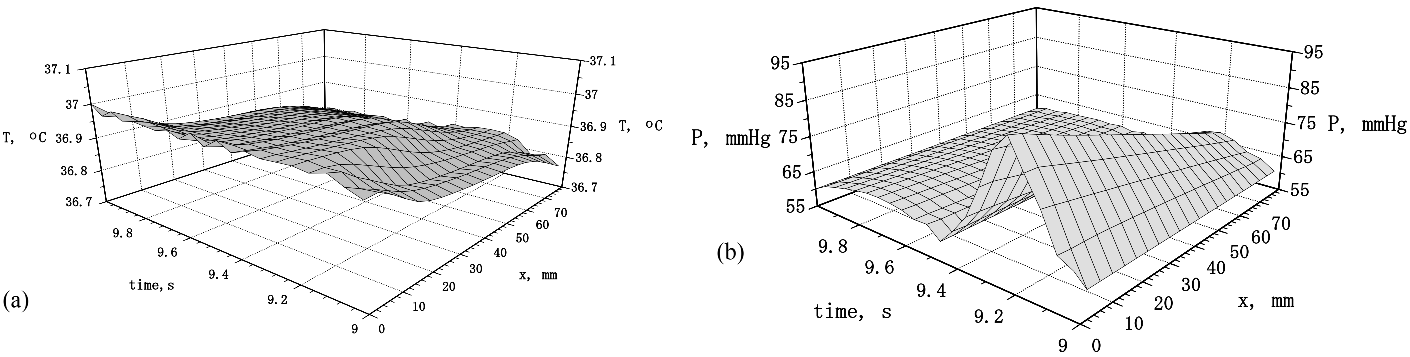

Figure 4. Temperature (a) and pressure (b) of the artery
as functions of x and t during one period

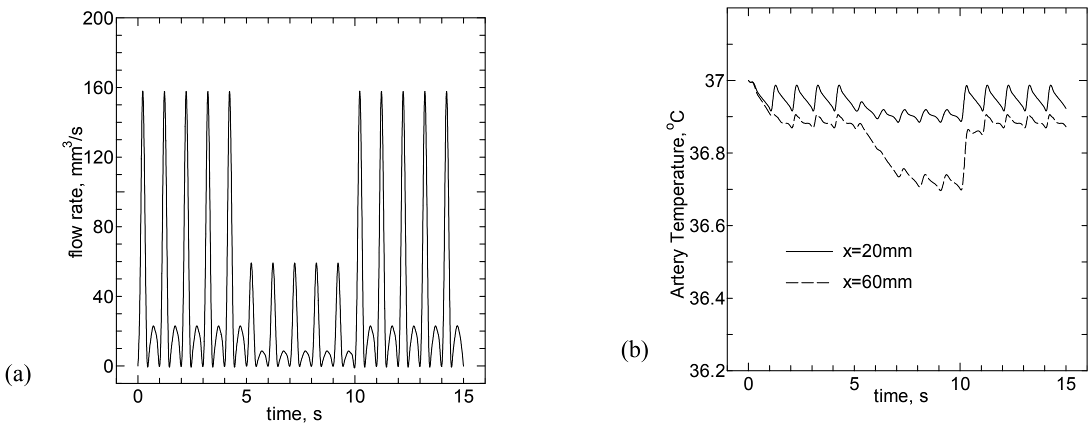

Figure 5. Temporal variations of the flow rate (a) and
arterial temperature (b) in the transient state

the arterial cross-sectional area being considered in the deduced energy.

This has been an initial study for investigating the effect of blood flow on the temperature response. In further studies, we intend to build up a computer model for the whole human systemic circulation that includes the systemic arterial circulation and the venous return flow circulation. It is hoped that the model can simulate the temperature variation in different transient states.

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