心血管系１次元数値シミュレーションモデルの高精度化

The establishment of high-precise one-dimensional numerical simulation model of blood flow for the cardiovascular system.

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Outline

To make a computational model of whole body circulatory system

Modify at junction point

3-D model:
- realistic
- 3D, pulsatile flow fields
- complicated and difficult

1-D model:
- idealized
- wave propagation
- feasible

Boundary conditions for 3-D model

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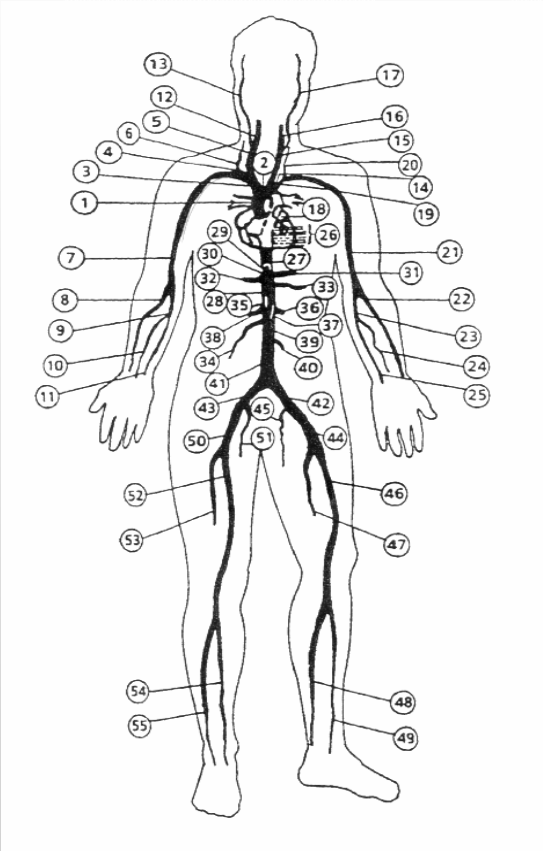
Objective

1-D whole body model



Quantitative model

Influence of some issues

- **Vessel structure (taper, branch, etc.)**
- **Unsteadiness of blood flow**
- **Behavior of vessel wall**
- **Boundary conditions**
- **Non-newtonian characteristics of blood**

Aim:

To establish the high-precise One-Dimensional numerical simulation model

Models:

- **Branch angle model**
- **Unsteady viscous model**
- **Generalized Viscoelastic Model**

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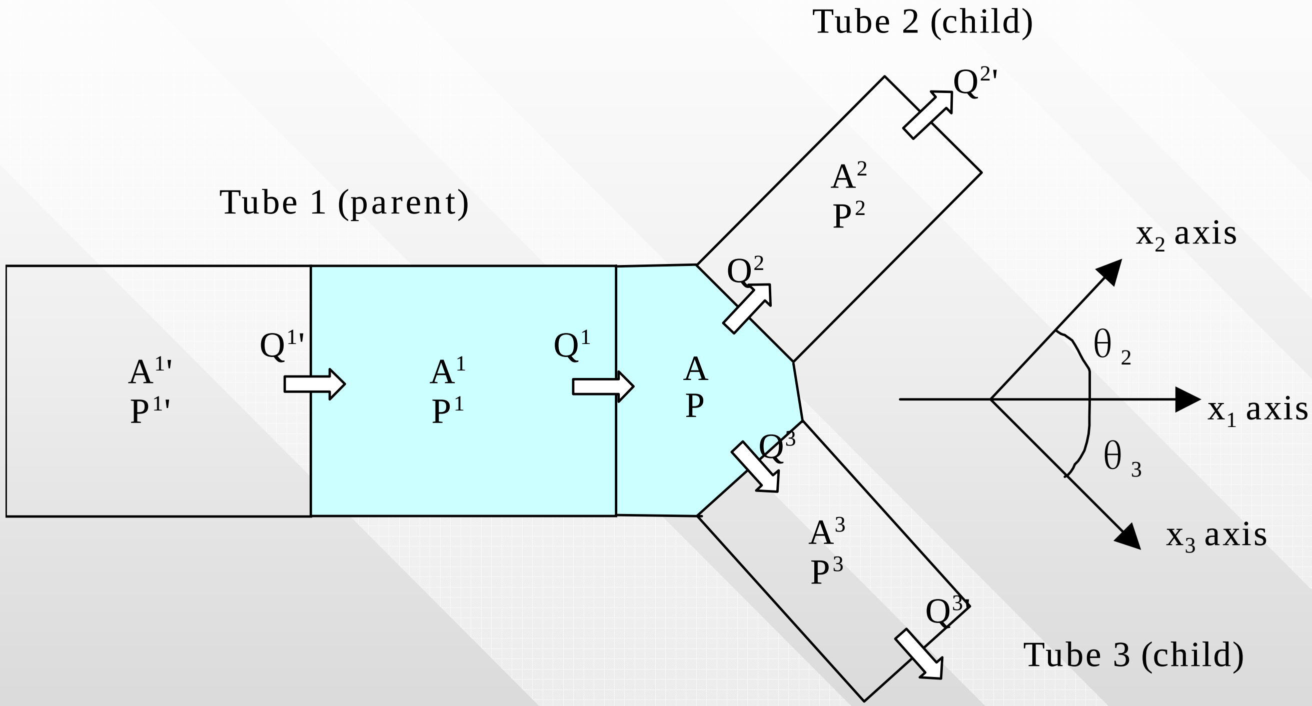
1: Branch angle

Treatment at branching points



$$\frac{dQ^1}{dt} = \left\{ \frac{AA^1}{A^1 + A} \left(\frac{(Q^{1'})^2}{(A^{1'} + A^1)/2} - \frac{(Q^2)^2}{AA^2} \cos\theta_2 - \frac{(Q^3)^2}{AA^3} \cos\theta_3 \right) + (P^1 - P)AA^1/\rho \right\} / \Delta x$$

$$AA^1 = \left(\frac{2A^1k^1}{A^1k^1 + A^2k^2 + A^3k^3} A + A^1 \right) / 2$$

cross-sectional area ratio of the tubes : $\dfrac{\sum A_{output}}{A_{input}}$

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Relationship between the reflected wave and the tube cross-sectional ratio

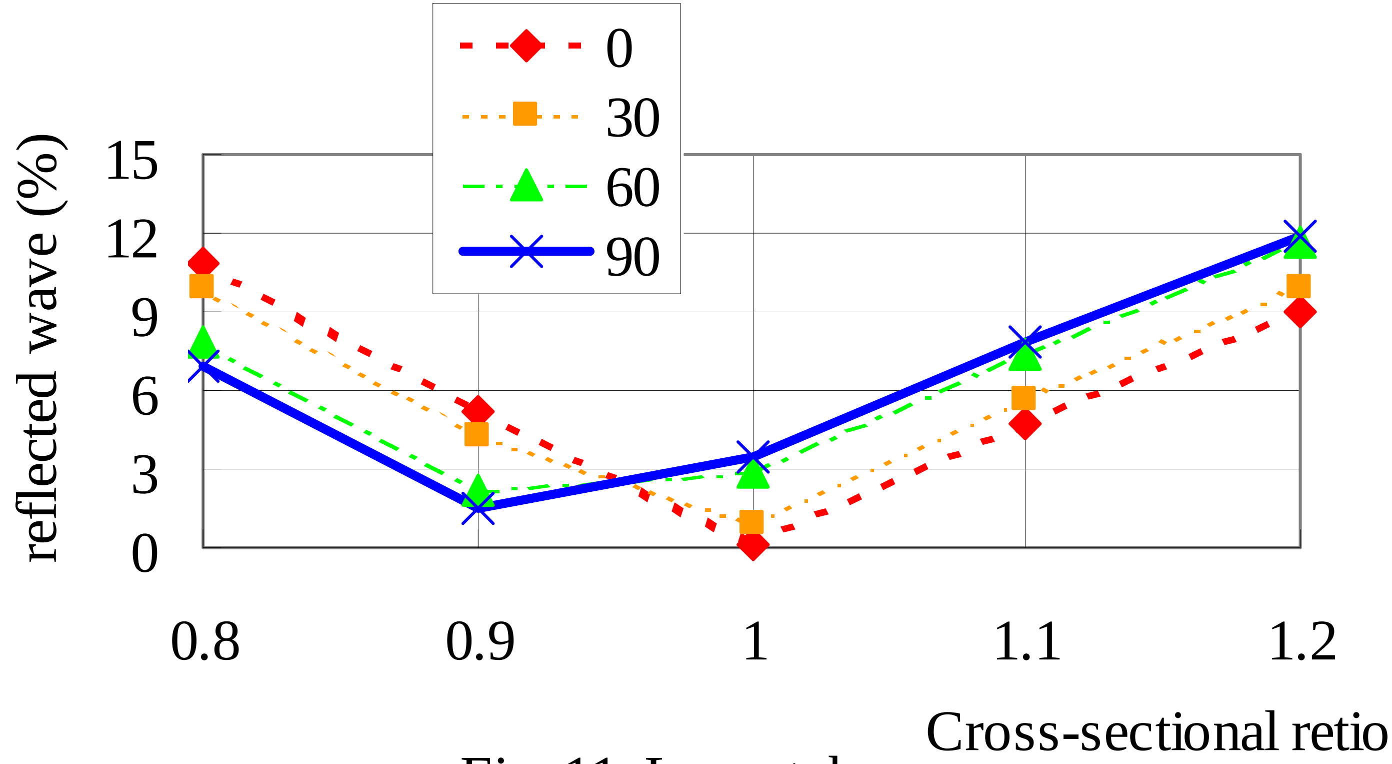


Fig. 11: Large tube

Discussion and Conclusion

- **1-D computational model of the artery systems**
  - investigation the bifurcation angle dependence
  - a quantitative analysis of the reflected wave
- **The angle effect**
  - the reflected wave at bifurcation point was observed
  - the angle dependence was recognized in large and medium arteries
- **Combination of angle and cross-sectional ratio**
  - peculiar feature of reflected wave

2: Unsteadiness of blood flow
3: Behavior of the vessel wall

One-Dimensional Numerical Model

- **Continuity equation**

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0$$

- **Equation of momentum conservation**

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+\frac{A}{\rho}\frac{\partial p}{\partial x}+f_t=0$$

unsteady viscous term

quasi-steady flow model

$$f_t=8\pi\nu V(t)$$

approximated unsteady model

$$f_t=c_v 8\pi\nu\frac{Q(t)}{A(t)}+(c_u-1)\frac{\partial Q}{\partial t}$$

accurate unsteady model

$$f_t=4\pi\nu\left\{2V(t)+\int_0^t W(t-u)\frac{\partial V(u)}{\partial t}du\right\}$$

- **Deformation law of the tube**

$$p-p_0=\frac{1}{Cs}(A-A_0)+f\left(\frac{\partial A}{\partial t}\right)$$

viscoelasticity of tube wall

Elastic model

$$f=0$$

Voigt model

$$f=\frac{hE_i}{2RA_0}\frac{\partial A(u)}{\partial t}$$

Generalized Viscoelastic Model

$$f=\frac{h}{2RA_0}\int_0^t\sum_{i=1}^{n}E_i e^{-(t-u)/\tau_i}\frac{\partial A(u)}{\partial t}du$$

Calculation methods of Generalized Viscoelastic Model

■ Unsteady Viscous term : *Kagawa et al.(1983)*

$$f_t = 4\pi\nu\left\{2V(t) + \int_0^t W(t-u)\frac{\partial V(u)}{\partial t}du\right\}$$

$$W(t) = \sum_{i=0}^{k} m_i e^{-n_i(\nu t)/R^2} \qquad y_i = \int_0^t m_i e^{-n_i(\nu/R^2)(t-u)}\frac{\partial V(u)}{\partial t}du$$

$$f_t(t) = 4\pi\nu\left\{2V(t) + \sum_{i=0}^{k} y_i(t)\right\}$$

$$\begin{cases} y_i(t) = 0 & (t=0) \\ y_i(t+\Delta t) = e^{-n_i(\nu\Delta t)/R^2} y_i(t) + m_i e^{-n_i(\nu\Delta t/2)R^2}\{V(t+\Delta t) - V(t)\} & (t>0) \end{cases}$$

■ Viscoelasticity of the tube

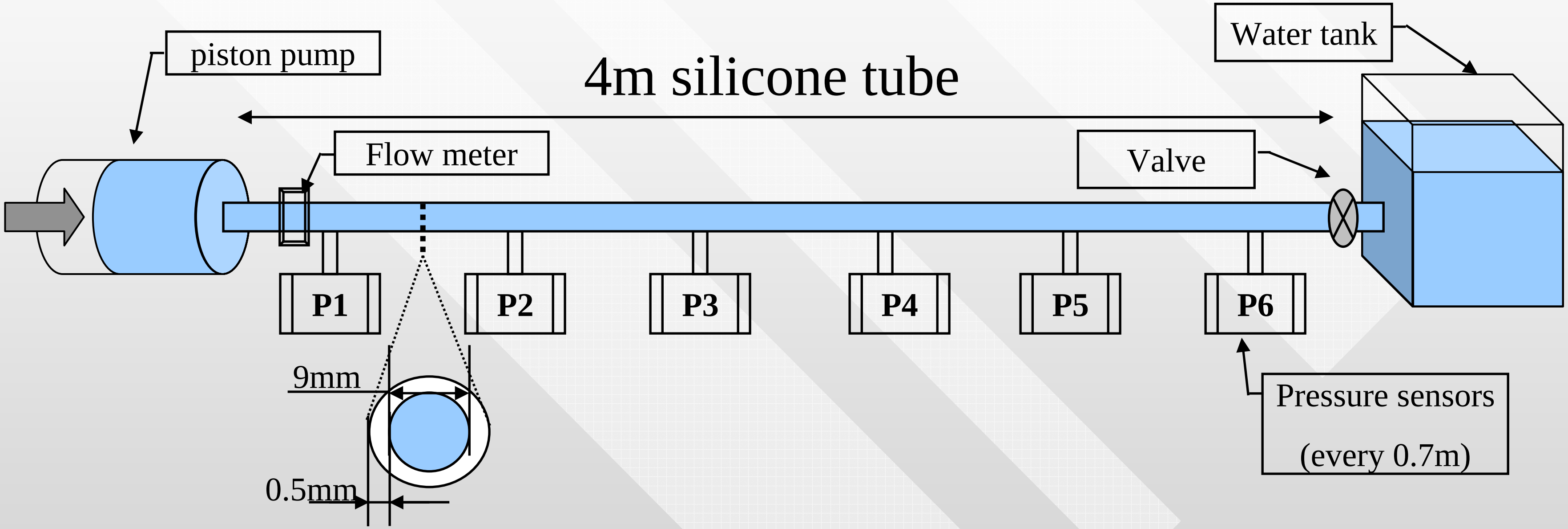
$$f = \frac{h}{2RA_0}\int_0^t \sum_{i=1}^{n} E_i e^{-(t-u)/\tau_i}\frac{\partial A(u)}{\partial t}du$$

$$Wv(t) = \sum_{i=1}^{n} E_i e^{-t/\tau_i} \qquad z_i = \int_0^t E_i e^{-(t-u)/\tau_i}\frac{\partial A(u)}{\partial t}du$$

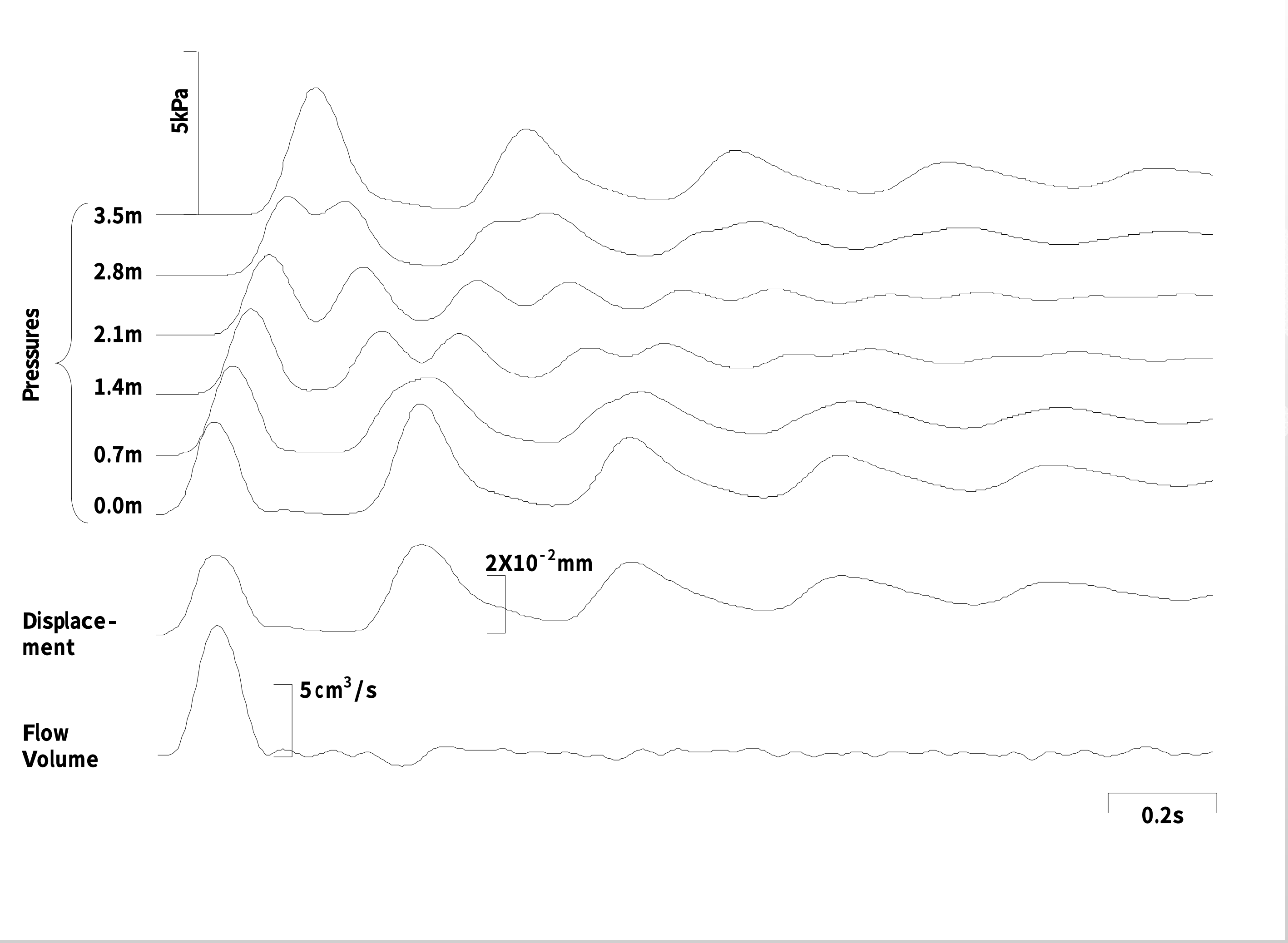
$$f = \frac{h}{2RA_0}\sum_{i=1}^{n} z_i(t)$$

$$\begin{cases} z_i(t) = 0 & (t=0) \\ z_i(t+\Delta t) = e^{-\Delta\tau/\tau_i} z_i(t) + E_i e^{-(\Delta\tau/2)/\tau_i}\{A(t+\Delta t) - A(t)\} & (t>0) \end{cases}$$

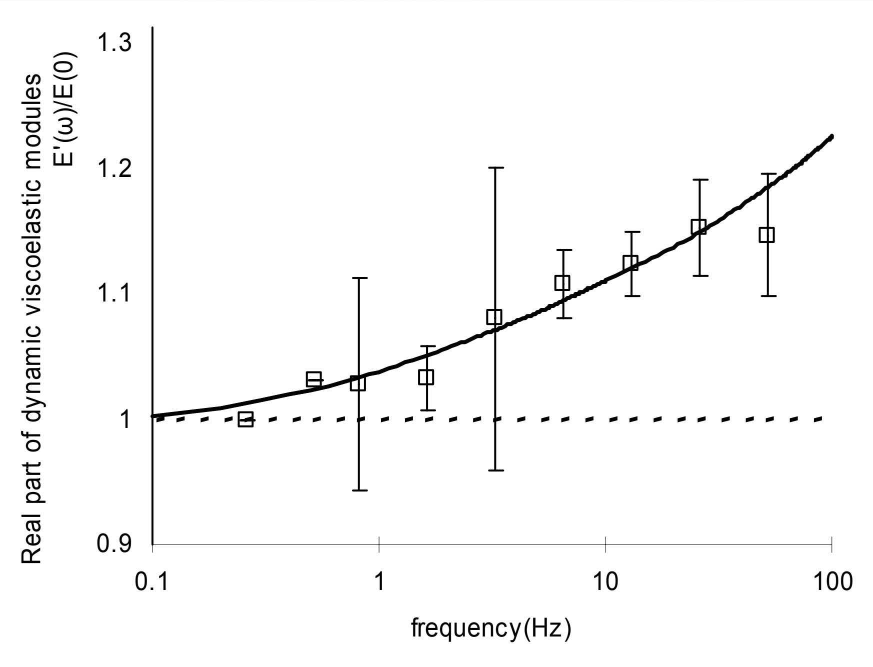
Experimental apparatus



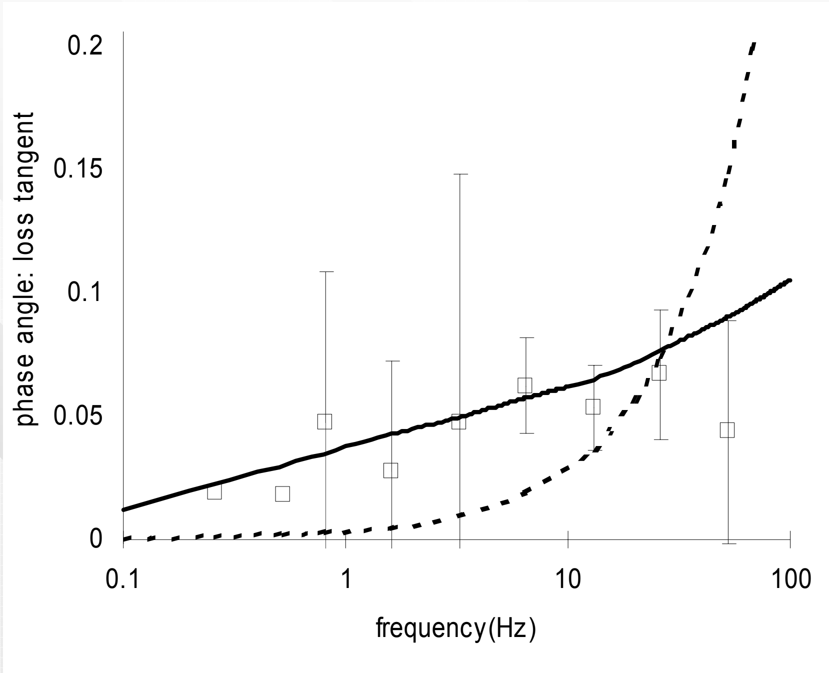
Experimental result



Determination of tube viscoelastic parameter



(a) real part of viscoelastic modulus



(b) loss tangent

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A computational method

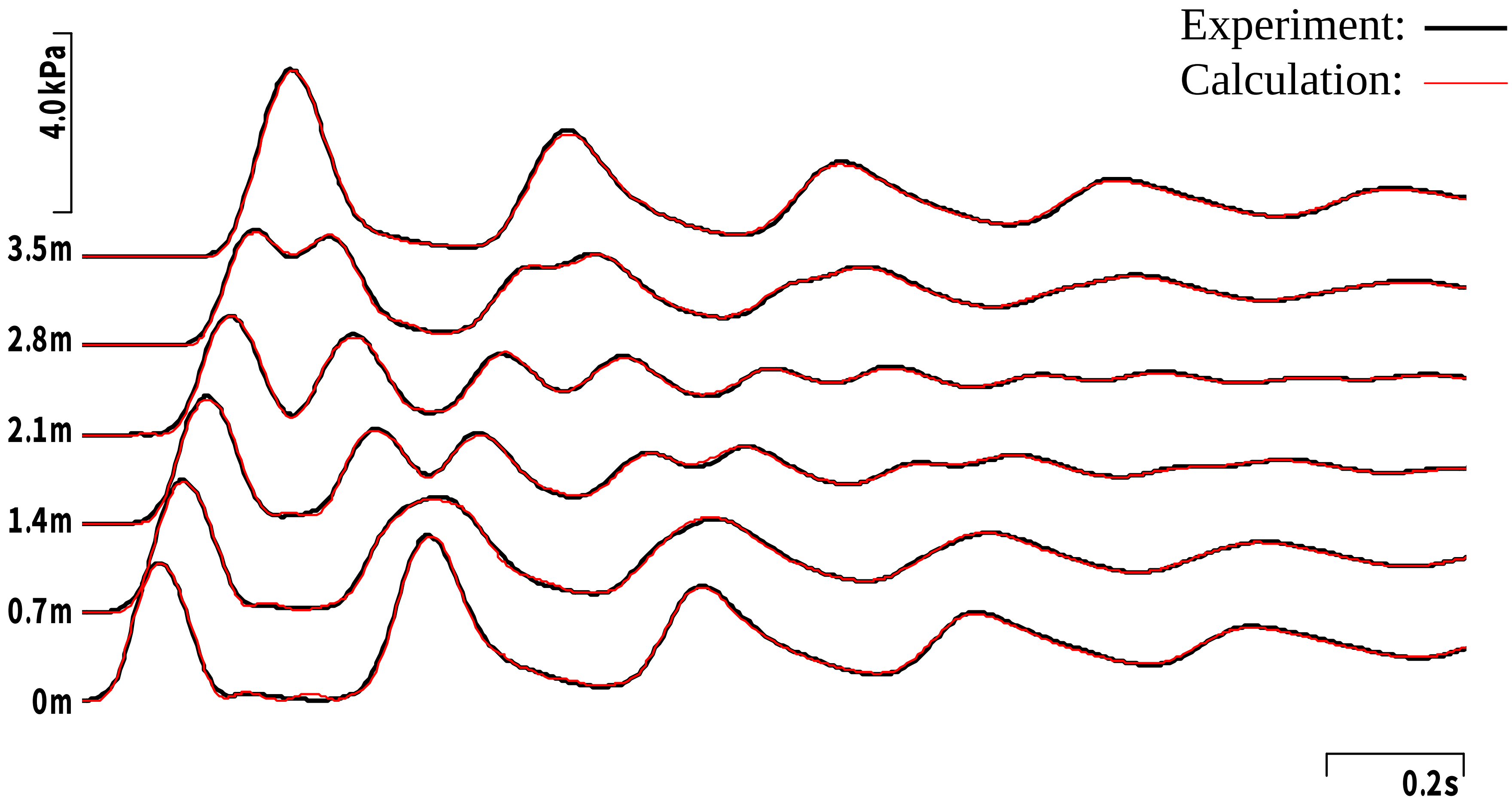
- **Computational scheme**
  - Finite Difference Method
  - Space : 4th order central difference
  - Time : Jemson-Baker four stage Runge-Kutta
- **Boundary conditions**
  - input:
    Flow volume
  - output:
    No output flow
- **Initial state**
  - no flow in the tube

Cross-sectional area (tube diameter)	0.612×10^{-4} m^2 (8.83 mm)
Input peak pressure peak flow rate	1.5 kPa (0.13 m/s)
Max Reynolds number (Re)	1150
Static Young module (E_0) (wave propagation velocity)	3.05 (MPa) (21 m/s)
Length of the tube (Δx)	4.0 m (0.05 m)
Total elapsed time (Δt)	4.0 s (0.001 s)
Courant Number (=c△t/△x)	0.42

Computational parameters

Comparison between measurement and simulation

Generalized Viscoelastic Model



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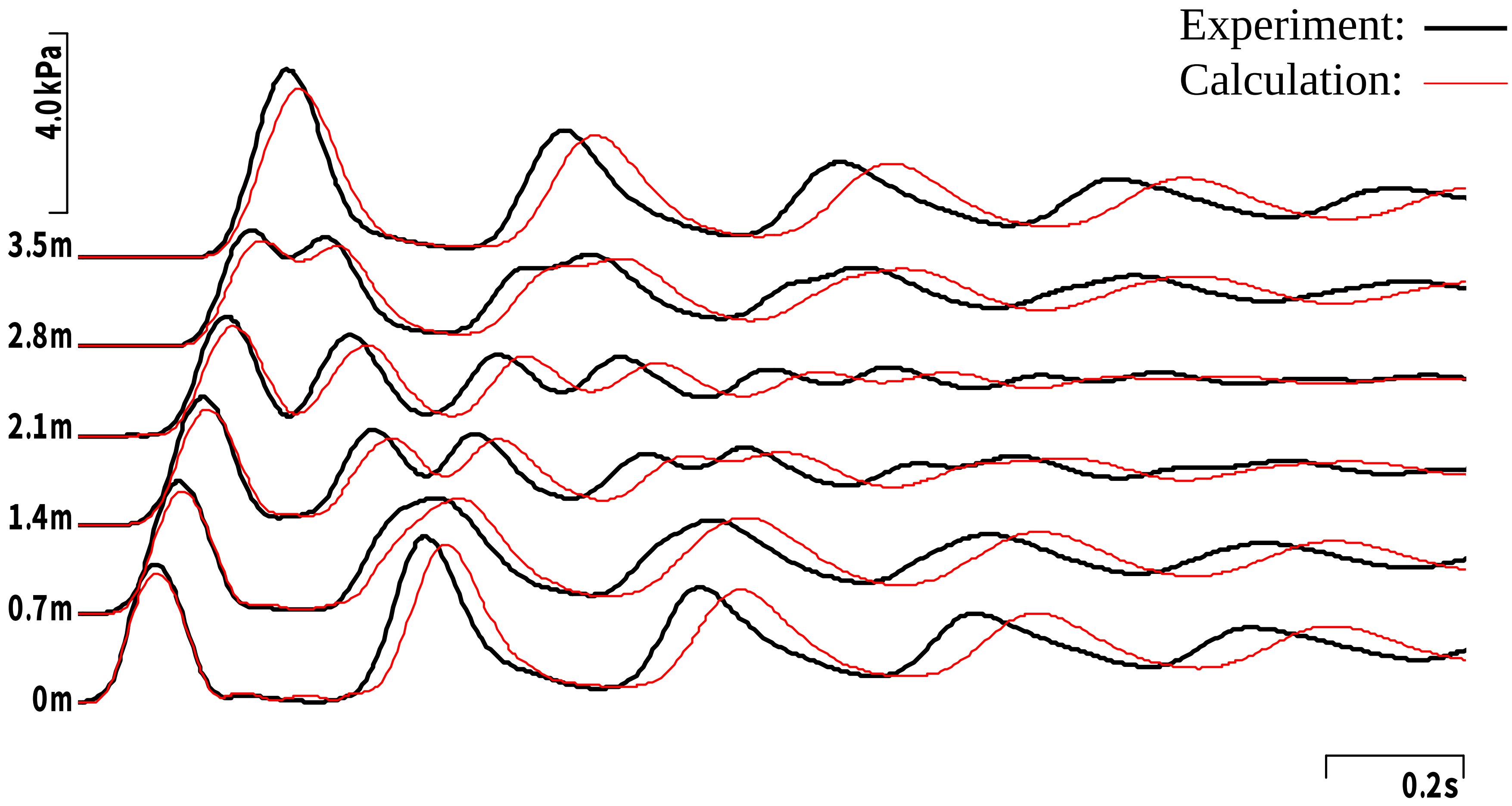
Comparison between measurement and simulation

Voigt Model



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Conclusion

- **Establishment the treatment of unsteady viscous term and vessel wall viscoelastic term**
  - **Unsteady viscous model and Generalized Viscoelastic Model can be applied to the deformable tube**
  - **New calculation method is established.**
  - **Good agreement with measurement and simulation involving both <u>unsteadiness</u> and <u>visco-elasticity of tube</u>**

Future works

- **Establishment of the whole body 1-D model**
  - **Decision of parameters: viscoelasticity of vessel wall**
  - **Apply to the *in vivo* phenomenon analysis**
- **Model combination**
  - **Tree-structured 1-D model and 3-D model**
- **Verification and validation**
  - **comparison with 3-D model, experimental results**