1-DIMENSIONAL NUMERICAL ANALYSIS OF BLOOD FLOW IN MULTI-BRANCHED ARTERIES

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Abstract.

Outline: *A numerical simulation study was carried out to clarify the relationship between changes to the physiological characteristic of the circulatory system and the change of pulse wave pattern.*

Content: *A one-dimensional simulation model with a multi-branch structure was constructed of the circulatory system. This model is different from the conventional circulatory system model in that the bifurcation angle can be examined*

Result: *Bifurcation angle had a small effect on the pulse wave propagation that affected the result of a quantitative analysis of the reflected wave.*

1. INTRODUCTION

Flow simulation is useful for understanding and quantifying fluid phenomena which arise in the systemic arteries.[(1)-(3)] However, these phenomena are complicated in vivo because of the following effect:

- Unsteady blood flow
- Non-Newtonian characteristics of the blood
- A change in the blood vessel geometry structure
- A change in the physiological characteristics
- A change in the heart output conditions

There are two types of model for explaining such phenomenon. The first is the one-dimensional fluid model (1-D model),[(4),(5)] which analyzes the change in flow along the blood vessel. The other is the two-dimensional[(6)] or three-dimensional model[(7)] which can reproduce the complicated shape of the blood vessels. Recent advances in computer technology have provided the means to analyze such a complicated shape. However, these models aim to clarify the local flow in narrow segments, bifurcations, etc., and do not reduce the systemic artery dynamics of the whole-body, because the whole-body analysis using the three-dimensional model is difficult even in present computer ability. Therefore, in order to carry out a quantitative analysis of the pressure pulse wave which is propagated in vivo, it is important to examine the quantitativity of the 1-D simulation model. Also, it is possible to use the result of the 1-D model as a boundary condition of the three-dimensional model. Previous 1-D models[(4),(5)] have not allowed for the effect of the angle of the bifurcation, although bifurcation of the arteries has various angles and radii,[(8)] that influence flow changes by the effect of the inertia term. A one-dimensional model with a variable bifurcation angle was therefore constructed for a numerical simulation of the circulatory system.

2. Numerical analysis model

2.1 Nomenclature

Q : flow volume
P : pressure
A : cross-sectional area of the tube
x : axial distance coordinate
t : time
ρ : density of the fluid
μ : viscosity coefficient of fluid
A_0: cross-sectional area with transmural pressure P_0
K : coefficient relating pressure and cross-sectional area

2.2 Construction of the 1-D model

The one-dimensional simulation model with a multi-branch structure to represent the systemic arteries was constructed according to the accepted fundamental equation. [4]

(1) Fundamental equation

The following were made:

- One-dimensional viscous flow disregarding the radial direction flow and secondary flow in the tube
- Laminar and steady flow in the tube (Poseuille flow) and neglects the effect of unsteady viscosity
- Normal wall deformation due to the usual blood pressure change without considering any large deformation and visco-elasticity of the vessel wall

According to these assumptions, the following fundamental equations were obtained:

- Equation of continuity

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

- Equation of momentum conservation

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{A}{r}\frac{\partial p}{\partial x} + \frac{8pm}{r}\frac{Q}{A} = 0$$

- Deformation of the tube

$$p = p_0 \exp\left(\frac{1}{K}\left(\frac{A}{A_0} - 1\right)\right)$$

(2) Numerical calculation method

A staggered lattice system was adopted, the space difference was established by the finite volume method, and an ordinary differential equation with time was obtained. This differential equation was solved by the 4-stage Runge-Kutta method, also called the Jemeson-Baker method,[9] with fourth-order accuracy.

2.3 Treatment of branching points

(1) Branching point

- The bifurcation involved one parent tube connected with two child tubes, with the grid geometry and physical quantity shown in Fig. 1.

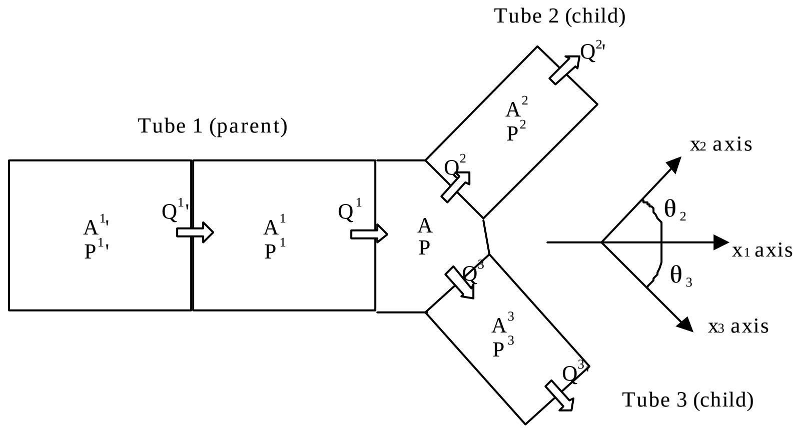

Fig. 1 Branching point geometry

A may satisfies the relationship V=A×△x, where V is the volume of the junction area.

(2) Unknowns

There are a total of 11 unknowns in the bifurcation region shown in Fig. 1: cross-sectional area (A, A^1, A^2 and A^3); Pressure (P, P^1, P^2 and P^3); Flow volume (Q^1, Q^2 and Q^3).

(3) Relationship between unknowns

- The ordinary differential equation with time was discretized as shown next.
- The 11 unknowns were solved From 11 relational expressions.

①Equation of continuity

$dA^1/dt = (Q^{1'} - Q^1)/\triangle x$, $dA/dt = (Q^1 - Q^2 - Q^3)/\triangle x$

with dA^2/dt and dA/dt expressed in a similar way

②Equation of momentum conservation

$$\frac{dQ^1}{dt} = \left\{ \frac{AA^1}{A^1 + A} \left(\frac{(Q^{1'})^2}{(A^{1'} + A^1)/2} - \frac{(Q^2)^2}{AA^2} \cos\theta_2 - \frac{(Q^3)^2}{AA^3} \cos\theta_3 \right) + (P^1 - P) AA^1/\rho \right\} / \Delta x$$

provided $AA^1 = \left(\frac{2A^1k^1}{A^1k^1 + A^2k^2 + A^3k^3} A + A^1 \right)/2$ (see the appendix)

dQ^2/dt and dQ^3/dt were expressed in a similar way

③Deformation of the tube

$P^1 = P^1(A^1)$, $P^2 = P^2(A^2)$, $P^3 = P^3(A^3)$ and $P = P(A)$

3. Calculated Results

3.1 Calculation model

(1) Model geometry

In present examination, a junction having one parent tube and two child tubes of equal cross-sectional area was considered as the simplest model. In order to examine the angle dependence of the reflected wave, the same bifurcation angle for the two child tubes was used. The effect of the viscous term was disregarded in order to quantitatively analyze the amplitude of the reflected wave.

(2) Calculation parameters

The common calculation parameters are shown in Table 1. The coefficient relating the pressure and cross-sectional area has been determined, when the pressure wave is small, to give a 5m/s～12.5m pressure wave propagation velocity. It is possible in this condition for the pulse wave reflected in the bifurcation to be analyzed separately from the input wave. In the model, the following three parameters were varied. The overall length of the parent and child tubes was 2.0m ～4.0m.

parameter	
density of fluid(ρ)	1.0×10^{3} m
viscosity of fluid(μ)	3.0×10^{-3} m^3/s

Table 1

i) The diameter of the tube

Three tube diameters were used to model blood vessel of the aorta (thick tube), middle artery (medium-thickness tube) and arteriole (thin tube) as shown in Table 2.

ii) The bifurcation angle

This angle was varied from 0° to 120° at 30° intervals.

iii) The cross-sectional area ratio of the tubes

The cross-sectional area ratio of child tube to the parent tube was varied from 0.8 to 1.2 at 0.1 intervals.

diameter	*thick*	*medium*	*thin*
Cross-sectional area (tube diameter)	5.0×10^{-4} m (25mm)	7.0×10^{-6} m (3mm)	2.0×10^{-7} m (0.5mm)
Maximum Flow Volume (q_0) (peak flow velocity)	5.0×10^{-4} m^3/s (100cm/s)	5.0×10^{-6} m^3/s (70cm/s)	3.0×10^{-8} m^3/s (15cm/s)
The relation coefficient (K) (wave propagation velocity)	4.0 (5.0 m/s)	10.0 (7.9 m/s)	25.0 (12.5 m/s)
Length of the model ($\triangle$x)	1.0m (1.0mm)	1.5m (2.0mm)	2.0m (5.0mm)
Total time of the model ($\triangle$t)	0.5s (0.1ms)	0.5s (0.1ms)	0.5s (0.2ms)
Courant Number (=c $\triangle$t/$\triangle$x)	0.5	0.395	0.5

Table 2

(3) Boundary conditions

The volume flow change shown in Fig. 2 were applied upstream of the parent tube and the reflection in the bifurcation was observed. The diameter of the parent tube governed the flow rate. The simple resistance model was used to provide the condition at the end of child tubes, although the effect of this end condition does not appears in this analysis which was completed before the pulse wave reached the tube end.

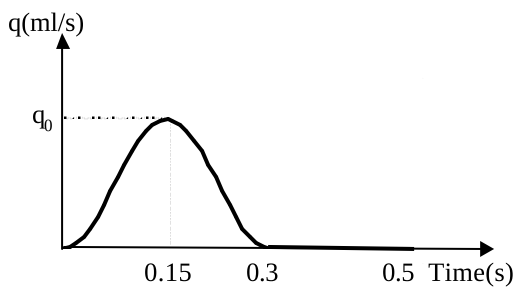

Fig. 2

(4) Calculation system

All calculations were carried out on a personal computer (CPU: Pentium III 500Mhz; Memory: 128MB; OS: Windows NT 4.0; Salford FORTRAN). $\triangle$t and $\triangle$x was set to sufficiently minimize the numerical error.

3.2 Calculated result

(1) Reflected wave at the branching point

Fig. 3 shows an example of the pressure-reflected wave (thin tube, 0° angle, and 1.2 cross-sectional ratio). In Fig. 4, part of the reflected wave shown in Fig. 3 has been expanded. The wave height shown in Fig. 4 defines the amplitude of the reflected wave when one was generated.

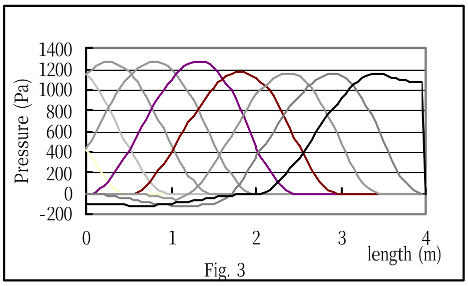

Fig. 3

(2) Relationship between the reflected wave and the tube diameter

The change in amplitude of the reflected wave of the blood vessel is shown in Fig. 5, 6 and 7. The horizontal axis is the vessel diameter, and the vertical axis is the proportion of the reflected wave to the input wave. In each figure, the amplitude change of the reflected wave with increasing bifurcation angle is shown. The changes in the reflected wave with tube cross-sectional ratios of 0.9, 1.0 and 1.1 are illustrated in Fig. 5, 6 and 7 respectively. No reflected wave was generated in the bifurcation for the case of the thin tube at 0° angle with a 1.0 cross-sectional ratio. However, a reflected wave generated, even with a 1.0 cross-sectional ratio, in the thick tube at 90° angle. A large reflected wave was generated due to impedance mismatching in all cases except with a 1.0 cross-sectional ratio. The results show that the reflected wave is not produced when the bifurcation angle is small, but that the reflected wave amplitude is changed as the bifurcation angle increases.

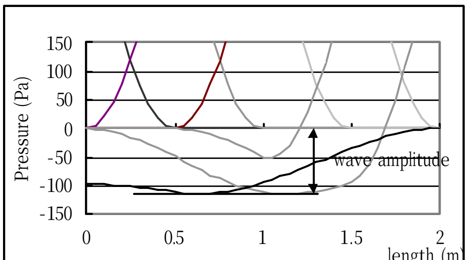

Fig. 4

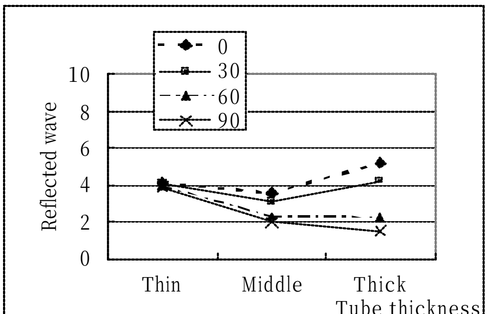

Fig. 5 : 0.9

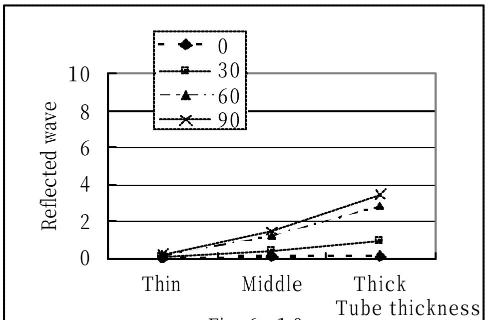

Fig. 6 : 1.0

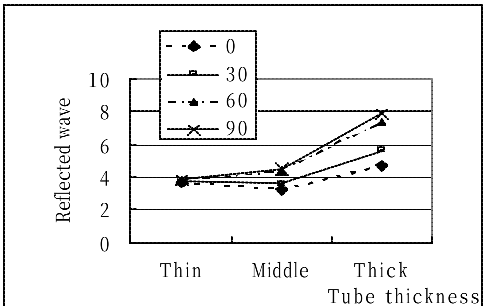

Fig. 7 : 1.1

(3) Dependence on bifurcation angle of the reflected wave

Fig. 8 shows the amplitude of the reflected wave with the cross-sectional ratio of the tubes set to 1.0 plotted against bifurcation angle. The effect of increasing bifurcation angle on the amplitude of the reflected wave is shown.

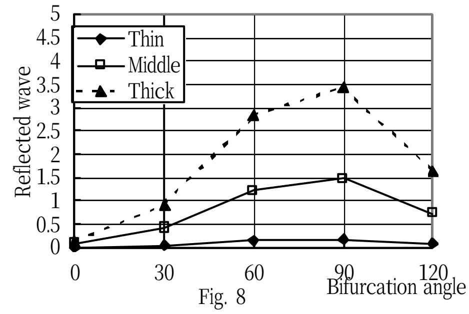

Fig. 8

(4) Relationship between the reflected wave and the tube cross-sectional ratio.

Fig. 9, 10 and 11 show the relationship between the cross-sectional ratio of the tube and the amplitude of the reflected wave plotted on the horizontal and vertical axes, for different tube diameters respectively. The results show that the bifurcation angle of the tube corresponds to about a 0.02 change in the cross-sectional area ratio.

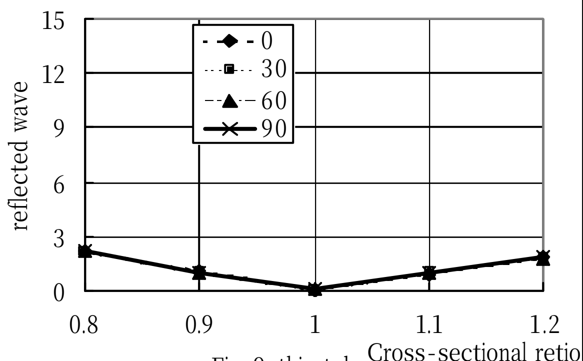

Fig. 9: thin tube

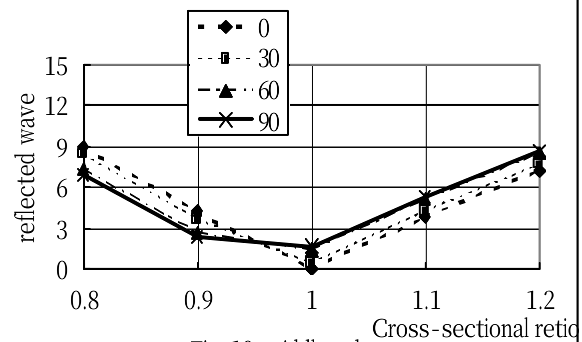

Fig. 10: middle tube

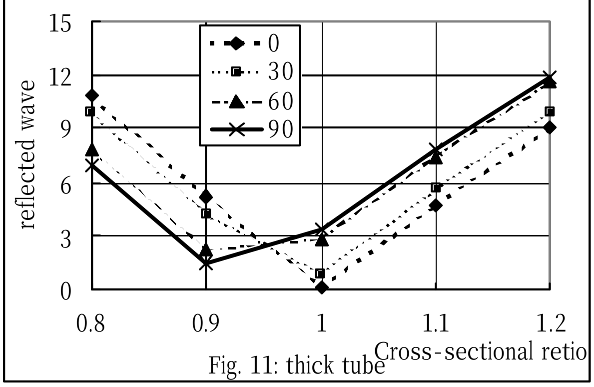

Fig. 11: thick tube

4. Discussion

• The angle effect of the amplitude of the reflected wave.

The reflected wave amplitude was about 3.4% of the input wave amplitude at a 90° of bifurcation angle (thick artery with a cross-sectional ratio of 1.0). This result shows that the bifurcation angle, which is not considered in the conventional 1-D simulation model, quantitativity affected the pulse wave propagation. Aortic arches have several junctions with branches at right angle to the main flow direction such as the carotid artery and arteria brachialis. The result of this simulation for the pulse wave at such a bifurcation, differ from those by the conventional 1-D simulation model. Although the effect is small the blood vessel bifurcation angle influences the pulse wave propagation. In particular, the effect on the reflected wave of the inertia term in the thick artery quantitativity influenced the pulse wave. However, the effect of such an inertia term is small and there is almost no quantitativity effect in arteries smaller than medium size.

・Relationship between the reflected wave and the cross-sectional ratio of the tube.
The reflected wave is greatly affected by impedance mismatching due to the cross-sectional ratio of the tube. In vivo blood vessels have various cross-sectional ratios, and it is also known that the value changes with the increasing age.[10] The change in amplitude of the reflected wave increased with increasing bifurcation angle and tube cross-sectional ratio.

The results of this study prove that an accurate quantitative assessment is difficult with the 1-D simulation which does not consider the bifurcation angle in the aorta. However, in blood vessels under artery size, since this angle dependence can be disregarded, the 1-D flow simulation is adequate. We therefore propose a three-level structure for simulating the arterial blood flow in the whole body:

- A model which accurately represents the three-dimensional structure of the arteries in the region where the flow rate is influenced by a change in the inertia term.
- The 1-D model of the blood vessel structure in the region where the bifurcation angle dependence is small.
- The structure tree model proposed by Olfsen[11] as the terminal condition for flow simulation in the region where the blood vessels are small and the terminal condition can not be measured.

This three-level model shows the three roles of the systemic arteries: the aorta (main transmission of blood), medium artery and arteriole (faucet to the capillary), and capillary (transmission of blood to the tissue). It is therefore a physiologically reasonable model.

5. Conclusion and Future Studies

A 1-D simulation model of the systemic arteries was constructed that considered the bifurcation angle dependence, and a quantitative analysis of the reflected wave in the bifurcation was made. It was proved that a reflected wave was generated, although it was small due to the effect of the inertia term in large arteries, with increasing bifurcation angle. A large reflected wave resulted when the cross-sectional ratio of the tube was large or small, and when there was the combined effect of a large bifurcation angle and cross-sectional ratio. The results of this study show the limitations of the conventional 1-D simulation to accurately model the effect of bifurcation angle in thick arteries when conducting a whole-body systemic analysis. We propose a three-level analysis structure to more accurately simulate the arterial system in vivo.

Future studies to expand on this work should address the following aspects:

- A comparison between the one-dimensional and three-dimensional model simulations.
- A comparison between the simulation result, data measured in vivo and experimental results from the structural model.
- An examination of the most effective simulation model combination.

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Appendix. Equation of momentum conservation at a bifurcation point

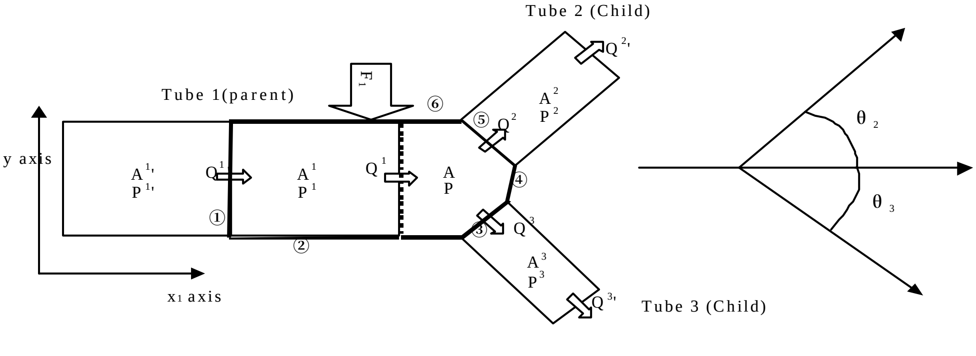

Fig : Geometry of a branching point

The momentum conservation law is considered in the x-axis direction. Frictional force at the tube wall is not considered.

The momentum in the region bounded by the bold line can be expressed by

$$mv = r\,(A^1\Delta x + A\Delta x)(Q^1 / AA^1 = r\,\Delta x Q^1\{(A^1 + A)/AA^1\}$$

where, AA_1 is the cross-sectional area through which Q_1 is flowing, and can be express by $AA^1 = (\frac{2A^1k^1}{A^1k^1 + A^2k^2 + A^3k^3} A + A^1)/2$.

This takes the average between A_1 and the A_1 component of A as the cross-sectional area through which Q_1 is flowing.

Similarly $AA^2 = (\frac{2A^2k^2}{A^1k^1 + A^2k^2 + A^3k^3} A + A^2)/2$ and $AA^3 = (\frac{2A^3k^3}{A^1k^1 + A^2k^2 + A^3k^3} A + A^3)/2$.

The momentum which flows through the region is $r\Delta t(Q^2 / A)$.

The total momentum flow becomes

$$r\Delta t(\frac{(Q^{1\prime})^2}{(A^{1\prime} + A^1)/2} - \frac{(Q^2)^2}{AA^2}\cos q_2 - \frac{(Q^3)^2}{AA^3}\cos q_3) .$$

The momentum term can be obtained as a difference equation from these two equations

$$\frac{dQ^1}{dt} = \frac{AA^1}{A^1 + A}(\frac{(Q^{1\prime})^2}{(A^{1\prime} + A^1)/2} - \frac{(Q^2)^2}{AA^2}\cos q_2 - \frac{(Q^3)^2}{AA^3}\cos q_3)/\Delta x$$

The pressure difference term at the Q_1 site is given by

$$(P^1 - P)AA^1 / r\Delta x .$$

The foregoing equations then allow the law of conservation of the momentum to be expressed as follows:

$$\frac{dQ^1}{dt} = \{\frac{AA^1}{A^1 + A}(\frac{(Q^{1\prime})^2}{(A^{1\prime} + A^1)/2} - \frac{(Q^2)^2}{AA^2}\cos q_2 - \frac{(Q^3)^2}{AA^3}\cos q_3) + (P^1 - P)AA^1/r\}/\Delta x \quad .$$